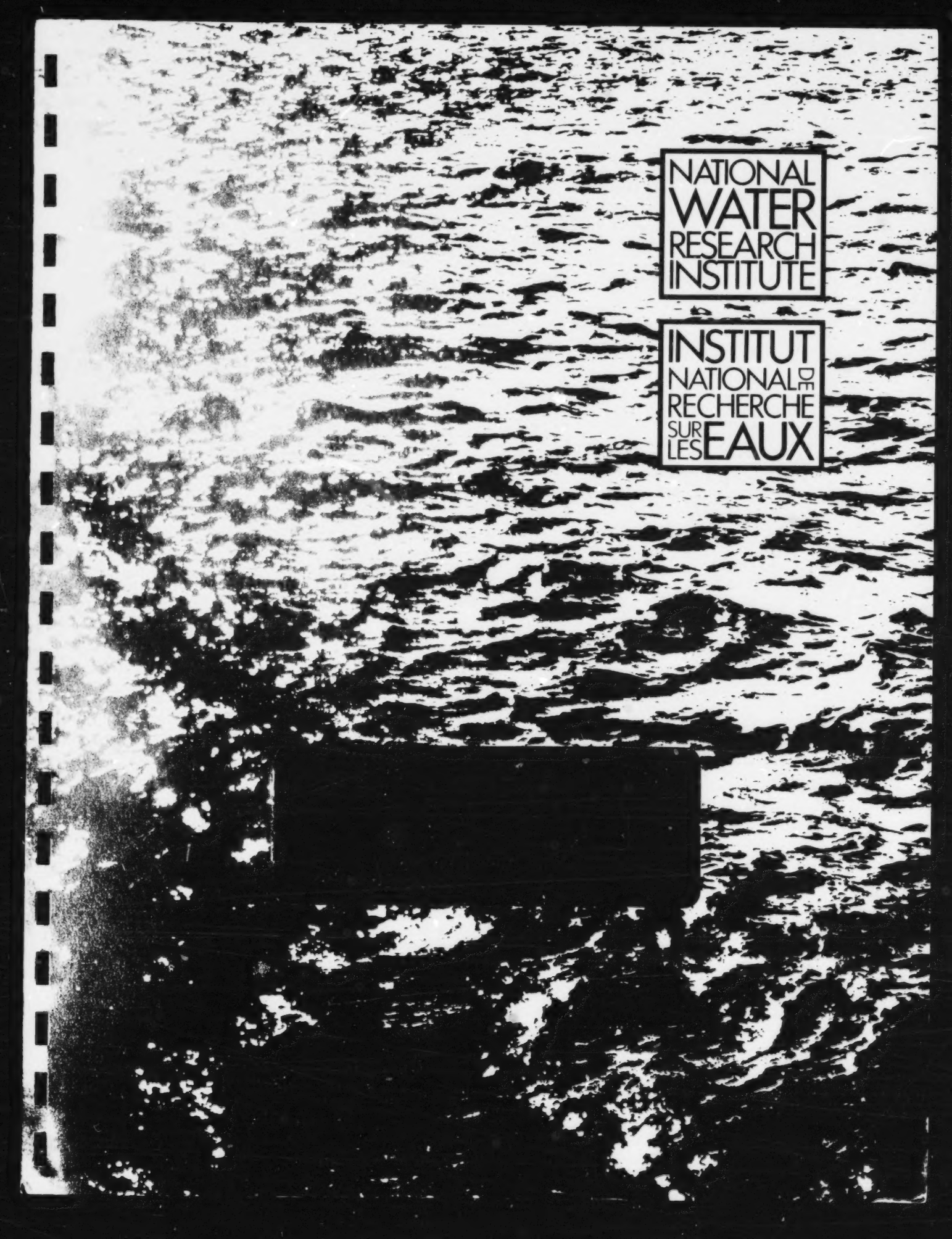
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# LABORATORY TESTS OF ARTIFICIAL CONTROL FOR THE MILK RIVER - PHASE II

by

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## MANAGEMENT PERSPECTIVE

Accurate discharge data are required for gauge sites such as the one on the Milk River at Eastern crossing so that the apportionment of water between Canada and the United States as dictated by the Boundary Waters Treaty can be satisfied. However, the streamflow records at this site are inaccurate because of unstable stage-discharge relationships caused by shifting channel bed and migrating sand waves.

This report examines the behaviour of a three dimensional weir selected from previous tests, known as the Flat Vee (FV) weir. Analytical considerations and data obtained from physical model tests indicate that the FV weir may be suitable for the Milk River at Eastern Crossing.

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## PERSPECTIVE - GESTION

Des données précises sur le débit doivent être recueillies dans des stations de jaugeage comme celle située sur la rivière Milk, à Eastern Crossing, de façon à respecter les dispositions relatives à répartition des eaux entre le Canada et les États-Unis du Traité sur les eaux limitrophes. Toutefois, les enregistrements du débit à cette station sont imprécis en raison des rapports variables entre le niveau de l'eau et le débit causés par le lit changeant du cours d'eau et les dunes hydrauliques mouvantes.

Le présent rapport examine le comportement d'un déversoir tridimensionnel choisi à partir d'essais précédents, et connu sous le nom de déversoir triangulaire. Les données analytiques obtenues d'essais sur maquette indique que ce type de déversoir pourrait convenir à la rivière Milk, à Eastern Crossing.

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## ABSTRACT

There is a need to improve the present method of obtaining the discharge at the hydrometric gauging station on the Milk River at Eastern Crossing, Montana. It has been decided to use a Flat Vee weir and tests in a sediment flume were conducted to assess the behaviour of this type of structure in a sand bed channel. The results were used to estimate the effective weir height above the stream bed for the prototype over the range of flow conditions to be encountered. It was concluded that the proposed weir at Eastern Crossing should function satisfactorily for most flows greater than 5 $m^3/s$. Additional tests for shallow approach channel conditions are recommended to reveal the behaviour of the discharge coefficient under such conditions. Tests to facilitate an efficient design for a stilling basin should also be conducted.

## RÉSUMÉ

Des améliorations doivent être apportées à la méthode de collecte de données sur le débit actuellement utilisée à la station hydrométrique de la rivière Milk, à Eastern Crossing, au Montana. Il a été décidé d'utiliser un déversoir triangulaire et des essais ont été faits pour évaluer l'efficacité de cet ouvrage dans un chenal à lit de sable. Les résultats ont permis d'estimer la hauteur utile du déversoir au dessus du lit du cours d'eau, pour divers débits. Il a été conclu que le déversoir proposé devrait bien fonctionner la plupart du temps lorsque le débit est supérieur à 5 $m^3/s$. En ce qui a trait aux chenaux peu profonds, d'autres essais devraient être faits pour évaluer les variations du coefficient de débit dans de telles conditions. Des essais en vue de la construction d'un bassin de tranquillisation devraient également être effectués.

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## 1.0 INTRODUCTION

Sandbed streams often present problems in determining their discharge because they are subject to changes in bed roughness and bed elevations due to scour and deposition. As a result, stage-discharge relationships are often poorly defined and corrections based on frequent water discharge measurements must be applied. The Milk River at Eastern Crossing, Montana, is an example of such a stream (Figure 1). The waters of the Milk River are shared by Canada and the United States in accordance with the Boundary Waters Treaty of 1919 and a 1921 Order of the International Joint Commission. Satisfactory apportioning of the water requires that reliable discharge records are obtained at the Eastern Crossing gauge.

Various methods to improve the measurement of discharge on the Milk River have been examined by Engel, Lau and Dick (1986) and Engel (1988). Requirements of good sensitivity at low flows, stability over the full flow range as well as good sediment and debris passing capability, indicate that a Flat-Vee (FV) weir as given by White (1970) and Bos (1976) would be suitable. However, there is some uncertainty regarding changes in bed level in the approach channel on the performance of the weir. The weir crest must be located above the channel bed by some minimum height at all flows and be free of sediment in order for the coefficient of discharge to be stable. To examine the effect of the sand bed in the approach channel, experiments in a sediment flume were conducted. The results are the subject of this report.

The work was conducted at the request of the Water Survey of Canada, Calgary, Alberta. All tests were carried out in the Hydraulics Laboratory of the Research and Applications Branch at the National Water Research Institute, Burlington, Ontario.

## 2.0 PHYSICAL CONDITONS AT EASTERN CROSSING

### 2.1 Flow Measuring Site

At the Eastern Crossing gauging station the river channel is straight for 200 m upstream and 800 m downstream from the present measuring cross-section. The flow enters this reach after passing through a sharp left bend (observer looking downstream) as shown in Figure 2. The bed of the river is composed of fine sand underlain by sandstone and coal. The right bank is much higher than the left bank and is not subject to overflowing. The left bank may overflow during very high floods or as a result of ice jams.

### 2.2 River Flow Variability

Flow measurement methods must take into account the flow variability as well as the flow range in order to achieve the greatest possible accuracy. In addition, because cross-sectional geometry of the river is dependent largely on the discharge, then flow variability becomes important also in selecting a method which is compatible with the expected behaviour of the river bed in the measuring reach.

To determine the flow variability, the 75 years of discharge records for the Milk River were examined. Monthly mean flows were plotted in Figure 3 for the month of March through October. This time period was selected because the records were complete during this time. The plot shows how the discharge is distributed over the seasons, indicating that the highest flows can be expected during the month of June. An indication of the daily variability can be obtained from Figure 4 which shows the hydrographs of daily mean flows for April, June and October in 1985.

The hydrographs show that the change in flow from day to day can be quite substantial as shown by the sharp peak for the month of June and the rapid rise in the flow during the month of April. Such rapid changes in the flow will result in accompanying rapid changes in the bed levels of the flow measuring reach.

Along with flow variablity, the selection of a method of flow measurement must also depend on the percent of time that a particular discharge is equalled or exceeded. The flow duration curve for daily mean flows, based on the 75 years of records at Eastern Crossing, given in Figure 5, shows that the median discharge is about 12 cms (cubic metres per second) whereas 10% of the time the flows are greater than 23 cms and smaller than 1.3 cms.

## 2.3 Changes in Cross-sectional Shape of the Channel

The channel bed goes through very dramatic changes with changes in discharge as shown for a very high flow and two flows somewhat above and below the average during the irrigation season (Figure 6). The plots show that at high flows of 83 cms the river scours its bank predominantly in the first 23 m from the right bank to a depth of about 2.1 m. When the discharge is about 20 cms, the scour along the right bank side is less with a tendency of increasing the depth toward the left bank. At a discharge of 14.5 cms the river bed cross-section has become almost uniform. The shape of the cross-section is in part related to the magnitude of the flow. At the highest flows the predominant scour toward the right bank reflects the effect of the sharp bend about 200 m upstream of the measuring section. The influence of the bend becomes progressively less as the flow decreases.

The change in the cross-sectional shape requires the movement of large quantities of sediment as can be seen from the super-imposed profiles in Figure 6. This variability in the cross-sectional shape and the bed elevations are the reasons why it has not been possible to establish a stable stage-discharge relationship at the Eastern Crossing gauge.

## 2.4 Sediment Loading

The variablity in the channel geometry is accomplished by the transport of large sediment loads carried by the flow. When the discharge is 30 cms, the suspended sediment transported per day is about 8000 tonnes. At 100 cms the sediment load would be equivalent to 86,000 tonnes per day. The suspended particle sizes range over three orders of magnitude, with the $D_{50}$ being always less than 0.1 mm. In contrast to the potentially large suspended load, the rate of bedload transport is much less. Records are virtually non existent but estimates based on three measurements made in 1978 (Spitzer, 1986) indicate that the transport rate is of the order of 10 tonnes/day when the discharge is 30 cms. The median particle size of the bedload as indicated from bed material samples are of the order of 0.1 mm to 0.2 mm.

# 3.0 ANALYTICAL CONSIDERATIONS

## 3.1 Properties of the Weir

The stage-discharge curve is used to determine discharge from measured values of the stage. It is therefore important to have a stable control which is not affected by sediment movement, bedform changes, deposition and scour. There is a wide range of flow measuring structures available, most of which have been developed for particular applications. Several artificial control structures were reviewed by Engel, Lau and Dick (1986). Results from preliminary model tests reported by Engel (1988) indicated that the most promising structure is the FV weir as shown schematically in Figure 7. The shallow Vee shape provides sensitivity at low flows and afflux at high flows is minimized because at these flows the full crest breadth is utilized.

In general, the discharge passing over the weir should depend on the geometric properties of the weir, the approach channel, the downstream channel and the fluid properties. These properties are reflected in the discharge coefficient which may be expressed in terms

of the upstream conditions and modular flow conditions (not submerged) in dimensionless form as

$$C = f(H/H_b, P/H_b) \quad (1)$$

where C = the discnarge coefficient, f denotes a function, H = the total energy head on the upstream side of the weir, $H_b$ = the difference in elevation between the lowest and highest point of the weir crest, P = the difference in elevation between the lowest point on the weir crest and the average surface level of the bed in the approach channel. For modular flow conditions White (1970) found that C varied only slightly for flows tested in the range of $0.4 < P/H_b < 7.0$. The coefficient of discharge is plotted in Figure 8 as a function of $H/H_b$ with $P/H_b$ as a parameter. The plot shows that the coefficient of discharge varies between the limits of 0.608 and 0.630 over the range of $P/H_b$ from 0.4 to 7.0. This variation in C is quite small and taking an average value of C = 0.620 will result in an error of the order of 2%. Such an error is acceptable considering the uncertainty in the stage-discharge relationship without a control structure.

## 3.2 Dimensional Analysis

The primary concern regarding the weir is the deposition of sediment in the approach channel. It is necessary to determine, if the conditions for which the discharge coefficient is stable, can be realized in a sand bed channel. To do this it is necessary to quantify the variability of $P/H_b$ in terms of the discharge passing the weir and the grain size of the sediment.

The height of the weir crest, P, above the mean bed level in the approach channel may be expressed in terms of the governing variables as

$$P = f(h_1, H_b, B, d, \rho, \nu, D_{50}, \gamma_s) \quad (2)$$

where $h_1$ = the measured head on the weir, B = the width of the approach channel, d = the length of reach of the approach channel measured from the weir crest, for which the average bed level is determined, $\rho$ = the density of water, $\nu$ = the kinematic viscosity of water, $D_{50}$ = the median grain size of the bed material in the approach channel, $\gamma_s$ = the submerged specific weight of the bed material and the other variables have been previously defined. The variables are shown schematically in Figure 7. Using dimensional analysis one obtains

$$P/H_b = f(h_1/H_b, d/H_b, B/H_b, D_{50}/H_b, \gamma_s D_{50}^3/\rho\nu^2) \quad (3)$$

For a given weir, $B/H_b$ is constant and if $d/H_b$ is also kept constant, then both of these variables can be dropped from equation (3). If one now writes $E = \gamma_s D_{50}^3/\rho\nu^2$, then equation (3) becomes

$$P/H_b = f(h_1/H_b, D_{50}/H_b, E) \quad (4)$$

If the fluid properties and the specific gravity of the sediment are kept constant, then each grain size represents a constant value of E. By keeping E constant, the effect of $h_1/H_b$ and $D_{50}/H_b$ can be revealed. The difficulty with solving equation (4) experimentally is that one cannot get a 1:1 correspondence between the model and the prototype values of $D_{50}/H_b$. This is because prototype values of $D_{50}$ are of the order of 0.1 mm to 0.2 mm and therefore values of $D_{50}$ in the model will become too small (Yalin, 1971). Unfortunately, this problem cannot be resolved with a distorted model (Engel, 1988) and therefore values of $P/H_b$ obtained from laboratory measurements must be scaled up with the aid of theoretical methods.

## 3.3 Theoretical Formulation

The maximum height of the weir, P, for a given discharge, is the distance between the weir crest at the apex of the Vee notch and the level of the bed at which scouring just ceases (i.e., the critical condition). Therefore, in order to determine $P/H_b$ it is necessary to solve simultaneously the equations governing the discharge passing over the weir and the critical conditions of the sand bed.

It has been shown by Yalin (1977), that the critical condition is given by the relationship

$$Y_c = \phi[E] \tag{5}$$

in which

$$Y_c = \rho U_{*c}^2 / \gamma_s D_{50}$$

where $U_{*c}$ = shear velocity at the critical condition, $\phi$ denotes a function and all other variables have been previously defined. Values of the critical mobility number $Y_c$ can be obtained for a given value of E from the curve in Figure 9 which is equivalent to the more commonly used Shields curve.

For the Milk River the median grain size for the bed material is of the order of 0.1 mm to 0.2 mm. As a result, the flows are in the smooth turbulent flow regime. For this type of flow the mean velocity for a two dimensional, uniform flow, as given by Yalin (1977), can be expressed as

$$\frac{U_m}{U_*} = 2.5 \ln\left[3.32 \frac{U_*(h_1 + P)}{\nu}\right] \tag{6}$$

where $U_m$ = the mean velocity of the flow. It is also known that

$$U_m = \frac{Q}{B(h_1+P)} \tag{7}$$

where Q = the discharge.

Therefore,

$$Q = U_m B(h_1 + P)$$

$$= 2.5\, U_* \ln\{3.32[EY_c]^{1/2}\, [\frac{h_1+P}{D_{50}}]\}B(h_1+P)$$

$$= 2.5\, (\frac{\gamma_s D_{50}}{\rho} Y_c)^{1/2} \ln\{3.32[EY_c]^{1/2}\, [\frac{h_1+P}{D_{50}}]\}B(h_1+P) \tag{8}$$

Given values for $h_1$ and $D_{50}$, E can be calculated and $Y_c$ can be obtained from Figure 9. If a value of P is assumed, then Equation 8 gives a value for the discharge. This value for Q can be compared with the value obtained from the following weir discharge equations given by White (1970).

$$Q = 0.4 \frac{B}{H_b} C\sqrt{g}\, H^{5/2} \qquad \text{for } H \leq H_b \tag{9}$$

$$Q = 0.4 \frac{B}{H_b} C\sqrt{g}\, [H^{5/2} - (H - H_b)^{5/2}] \qquad \text{for } H > H_b \tag{10}$$

and

$$H = h_1 + \frac{Q^2}{2gB^2(h_1+P)^2} \tag{11}$$

where H = the total energy head on the weir and C is a discharge coefficient which was obtained empirically. The correct value of P can be obtained by assuming different values until the discharge calculated

from Equation 8 for the critical shear stress is equal to the discharge calculated from the weir-discharge equations. From these results, the dimensionless variables $P/H_b$, $h_1/H_b$, and $D_{50}/H_b$ can be computed.

Values of $P/H_b$ were computed as a function of $D_{50}/H_b$ with $h_1/H_b$ as a parameter for a single value of E which reflects the use of the same value of $D_{50}$ in both model and prototype. The resulting curves are plotted in Figure 10. The curves show that $P/H_b$ is only mildly sensitive to changes in $D_{50}/H_b$ over the range from model to prototype for small values of $h_1/H_b$. As values of $h_1/H_b$ increase above 0.4 the effect of $D_{50}/H_b$ becomes increasingly significant. The curves in Figure 10 also show that values of $P/H_b$ for the prototype are negative for values of $h_1/H_b$ less than about 0.5, indicating that the elevation of the bed in the approach channel is higher than the elevation of the weir crest at the centre of the Vee notch. The FV weir will operate satisfactorily for values of P/H as low as 0.05 (Bos, 1976). The curves in Figure 10 indicate that this constraint is not met for values of $h_1/H_b < 0.6$ in the prototype. This means that the weir would be potentially non functional for flows up to about 15 cms. Flows between 5 cms and 25 cms are of primary interest and therefore values of $P/H_b$ in this flow range should never be less than 0.05.

The computed curves in Figure 10 are obtained by assuming two dimensional flow conditions in the approach channel. This is not quite true for the actual conditions because of the effect created by the the three-dimensional geometry of the weir. In addition, the critical shear stress condition at the bed is obtained from a curve equivalent to the Shield's curve which implies the existence of a plane bed. In reality the bed in the approach channel, after the cessation of bedload transport, is composed of ripples which are the bed forms for the value of the $D_{50}$ found at the Eastern Crossing prototype location. This means that the true values of $P/H_b$ can be expected to differ somewhat from the computed values in Figure 10. It is therefore necessary to conduct a set of experiments to compare the computed values of $P/H_b$ with values obtained from direct measurements in a model. Any significant

difference can be taken into account by adjusting the computed values in accordance with the measured values. Once this adjustment has been made for the model, final values of $P/H_b$ for the prototype weir can be determined.

## 4.0 EXPERIMENTS

The experiments were conducted in a flume especially constructed for this purpose, 20 m in length, 2.0 m in width and approximately 0.6 m in depth as shown in Figure 11. The flume was built directly on the laboratory floor and therefore its slope could not be adjusted. The flow depth was controlled by a set of vertical louvres located at the downstream end of the flume. Water was supplied to the flume from a large constant head tank, through a head box. The flow was conditioned prior to entering the test section by a series of baffles and flow straighteners. The water temperature was constant at about 18°C.

The weir was placed into the flume 15 m from the entrance on an elevated false floor about 0.30 m above the flume bed, thereby providing a recess into which the sediment was placed for a length of 8 m upstream from the weir. The weir was a typical 1:2/1:5 FV weir as given by White (1970) with the crest having a cross slope of 1:20, centred in the flume and extending over its full width of 2 m. The head was measured at the standard distance of $10H_b$ m upstream of the weir crest.

The mobile bed upstream of the weir consisted of a sand, similar to that of the Milk River at Eastern Crossing, having a median diameter of 0.22 mm. The grain size distribution of the test sand is given in Figure 12.

### 4.2 Measurement of Bed Level

At the start of an experiment the sand was levelled to obtain a smooth bed with a surface elevation equal to that of the lowest point

on the weir crest. The downstream louvres were closed and water was gradually introduced upstream, taking care that no sediment was transported over the weir at this time. This process was continued until the tailwater was well above the level of the apex of the vee notch. At this depth the flow rate could be increased without creating any scouring of the sediment in the approach channel. The discharge was increased and the louvres adjusted until the desired flow rate was obtained. The tailwater was then set so that the final flow conditions at the weir were modular (i.e., unsubmerged). Each experiment was allowed to continue until sediment movement in the approach channel had ceased. The head on the weir was then measured, the discharge was determined and the water was slowly drained from the flume, taking care that the scoured bed was not disturbed. Once the bed had sufficiently drained, cross-sectional profiles were taken at distances of $5H_b$, $10H_b$, $15H_b$ and $20H_b$ upstream from the crest, which corresponded to .25 m, .5 m, .75 m and 1.0 m respectively.

## 4.3 Measurement of Water Levels

The water levels for the determination of the head on the weir were measured using a stilling well fabricated from a 5 cm diameter acrylic cylinders fitted with Mitutoyo point gauges mounted on top as shown in Figure 13. The point gauge had a resolution of 0.05 mm. The intake of the stilling well was placed at a distance of $10H_b$ m upstream of the weir crest in the side of the flume in accordance with standards presented by White (1970). The stilling well was set to read zero at the elevation of the lowest point of the weir crest.

## 4.4 Measurement of Discharge

Prior to an experiment, the pump being used to provide the flow was rated to obtain the true pumping rate. Once a flow with the desired head over the weir crest had been established in the flume, the

discharge could be determined. This was done by diverting the overflow from the constant head tank into a calibrated volumetric tank in which it was measured by observing the change in volume over a measured interval of time. The discharge through the flume was then simply the difference between the flow rate supplied by the pump and the overflow rate of the constant head tank. This procedure was repeated twice for each experiment and the results averaged, providing an accuracy of about 2%.

### 4.5 Data Summary

A total of six experiments were conducted. The bed level of the approach channel near the weir was taken as the average elevation of the four profiles obtained for each flow condition and for each case the corresponding value of P was computed. The values of P for the six experiments conducted, along with the other necessary data are given in Table 1. Typical cross-sectional profiles at the distance of $10H_b$ upstream from the weir crest (i.e., along the line of the stilling well intake) are given in Figures 14 through 19.

## 5.0 RESULTS

### 5.1 Data Analysis

The experimental data from Table 1 were plotted as $P/H_b$ versus $h_1/H_b$ for the value of $D_{50}/H_b$ representing the model conditions in Figure 20. In addition, data from White (1971) were also plotted for the same value of E but having a $D_{50}/H_b$ only half as large as that used in the present tests. The plot shows that there seems to be no significant effect of $D_{50}/H_b$ over the range covered by the data. This differs from the trend exhibited by the theoretical curves in Figure 10. The curves in Figure 10 show that the effect of $D_{50}/H_b$ is very mild for values of $h_1/H_b$ less than 0.5 but becomes increasingly significant as $h_1/H_b$ increases from 0.5 to 1.5. Additional data from White (1971)

were plotted for the same value of $D_{50}/H_b$ but for a larger value of E to show the effect of this latter parameter. The plot shows that for a given $h_1/H_b$ and $D_{50}/H_b$ values of $P/H_b$ are larger for the data having the larger value of E. The reasons for this can be explained with the aid of Figure 9. For the bed materials being considered, the flow regime is in the upper smooth turbulent to lower transitional stage. Therefore when sediment size and specific gravity of the bed material combine to increase the value of E then the value of the critical mobility number is decreased which in turn means that the sediment is more easily eroded and a larger value of $P/H_b$ can be expected for a given flow condition. Because of this effect of E on the sand bed erodibility, it was necessary to use a sand size in the model tests which was closely similar to the prototype bed material in all respects.

## 5.2 Comparison of Predicted and Experimental Results

Values of $P/H_b$ were generated and plotted as a function of $h_1/H_b$ for the model value of $D_{50}/H_b$. A smooth curve was fitted through the points and this is given in Figure 21. The plot shows that $P/H_b$ decreases from a value of zero at $h_1/H_b = 0$ to a minimum value of -0.33 at $h_1/H_b = 0.6$ and thereafter increases as $h_1/H_b$ increases, taking on positive values when $h_1/H_b$ is greater than about 1.25. The negative values are the mathematical equivalent to a bed elevation above the elevation of the weir crest at the Vee notch.

To compare the predicted curve with the experimental measurements, the data in Figure 20 for E = 172 were also plotted in Figure 21 and a smooth curve fitted through the plotted points. The two curves show that the measured values of $P/H_b$ are always larger than the theoretical values. In addition, the rate of change of $P/H_b$ as $h_1/H_b$ increases is considerably less for the measured data showing that the sensitivity of $P/H_b$ to changes in $h_1/H_b$ is over predicted. The reason for these differences must be attributed to the geometry of the weir. As a result of the triangular cross sectional shape, the flow

tends to be concentrated toward the centre of the approach channel. This, together with the ripples and the complex flow patterns created in the near vicinity of the weir, tend to increase the scour of the bed, while at the same time dampening the rate of increase in the average cross sectional scour as the flow over the weir is increased.

The difference between the measured results and the predicted values of $P/H_b$ must be taken into account to arrive at the corresponding values of $P/H_b$ for the prototype weir.

## 5.3 Prototype Values of $P/H_b$

The differences between the predicted and measured values of $P/H_b$ were computed as $(\Delta P/H_b)$ and these are given in Table 2. The data in Table 2 were then plotted as $(\Delta P/H_b)$ vs $h_1/H_b$ in Figure 22 and a smooth curve was fitted to reflect the trend revealed by the plotted points. The curve shows that $(\Delta P/H_b)$ should increase from a value of 0 for $h_1/H_b = 0$ to a maximum value of about 0.4 when $h_1/H_b = 1.2$. Thereafter, values of $(\Delta P/H_b)$ steadily decrease as $h_1/H_b$ increases. If one assumes that this difference between the measured and predicted values of $P/H_b$ is not affected by changes in $D_{50}/H_b$ and is therefore the same for model and prototype, then the prototype values of $P/H_b$ can be determined from

$$(P/H_b)_p = (P/H_b)_c + \Delta P/H_b \qquad (12)$$

where subscript p refers to prototype and subscript c refers to the predicted value of $P/H_b$ at the prototype conditions. Values of $(P/H_b)_p$ according to equation (12) were computed and these are given in Table 3. $(P/H_b)_p$ was plotted as a function of $h_1/H_b$ in Figure 23. The plotted points were fited with a smooth curve, which shows that the minimum value of $(P/H_b)_p$ is about -0.1.

## 5.4 Effect of Reducing the Grain Size on Prototype Behaviour

It was shown in Figure 20 that an increase in the parameter E for a given value $D_{50}/H_b$ would result in an increase in the value of $P/H_b$. Similarly, one would expect a reduction in $P/H_b$ if E is reduced. Since the fluid in both model and prototype is water and the sediment is sand, any significant changes in E must be made by changing the grain size. The median grain size of the bed material at Eastern Crossing ranges between 0.1 mm to 0.2 mm. Theoretical values of $P/H_b$ were computed for $D_{50}$ = 0.22 mm and 0.15 mm and the results are given in Table 4. Since the weir size is kept constant, the values of $P/H_b$ are always larger for the smaller grain size, indicating that the results obtained with the aid of the model are conservative, as long as the median grain size in the prototype approach channel is smaller than that used in the model. The values of $P/H_b$ decrease in spite of the decrease in E because the value of $D_{50}/H_b$ in this case is not kept constant. By keeping the prototype value of $H_b$ constant, the reduction of the grain size from 0.22 mm to 0.15 mm, results in a reduction of $D_{50}/H_b$ from 0.000183 to 0.000125. Values of $P/H_b$ for $D_{50}/H_b$ = 0.000183 when $D_{50}$ = 0.15 mm are less than those obtained when $D_{50}$ = 0.22 mm. However, because $P/H_b$ increases as $D_{50}/H_b$ decreases, values of $P/H_b$ for the smaller grain size are larger when $D_{50}/H_b$ = 0.000125.

## 5.5 Implication for the Prototype Weir

The curve in Figure 22 shows that for values of $h_1/H_b < 0.4$ the elevation of the bed in the approach channel will be equal to or greater than the elevation of the lowest point on the weir crest. According to Bos (1976), a minimum value of $P/H_b > 0.05$ is required in order for the weir to operate satisfactorily. This condition is satisfied when $h_1/H_b > 0.43$ or when the discharge is greater than about 5 cms. However, the behaviour of the discharge coefficient needs

to be examined when values of $P/H_b$ become less than 0.4. It was shown in Figure 8 that the coefficient of discharge, C, is reduced when $P/H_b$ is decreased from 7.0 to 0.4. A further reduction in C can be expected when $P/H_b$ reaches values less than 0.4. The dependence of C on $P/H_b$ for $P/H_b < 0.4$ could not be determined during the present tests because of size limitations of the model. A larger model is required in order to avoid scale effects as values of $P/H_b$ approach the lower limit.

The values of $P/H_b$ can be expected to be dependent on the rate of sediment transport especially if the transport is greater than that due to the channel bed contributions alone. Under these circumstances the sediment near the weir and in the approach channel is replaced as fast as it is scoured away. Such a situation may occur during a recession after a flood flow or if sediment is introduced into the flow as a result of slumping of the river banks. At such times there may in fact be a net sediment build up in the approach channel particularly in the near vicinity of the weir itself. This means that the effective value of $P/H_b$ may be less than values occurring during flows with lower concentrations. The present tests were conducted under steady state conditions and do not reflect this. There is no reliable way to predict such unsteady conditions. However, considering the minimum value of $P/H_b$ at which the weir will still operate satisfactorily, it is expected that local flow patterns created by the weir itself will be effective enough to keep the weir sufficiently clear of sediment for measurement of all flows greater than about 5 cms.

As a result of deposition of excess sediment, White (1971) has found that shoaling may occur near the sides of the approach channel. Although these deposits do not affect flow over the weir itself, they tend to obstruct the stilling well intake. There is no reliable way of preventing the obstruction of the intakes and the best solution is to find an alternative means of measuring the stage of the flow.

## 5.6 Additional Considerations

The bed immediately downstream of the weir structure must be protected from flow conditions which may cause excessive scour which in turn could lead to undermining of the weir block. This is best done by incorporating a stilling basin in the weir design to encourage the dissipation of excess energy before the flow passes over the erodible bed below the weir. This is particularly true for the FV weir because flow intensity across its width is not constant. If the transverse velocity gradient is allowed to persist beyond the stilling basin, scour will be pronounced in the downstream sand bed channel. For this reason the stilling basin must act as a flow dispenser with the aim of producing a flat velocity gradient across the width of the downstream channel. White (1966) developed a preliminary design for a stilling basin, but further refinements regarding the transverse velocity distribution are necessary. Therefore, further tests are required to design the stilling basin. These tests should be conducted for both modular and submerged flow conditions.

# 6.0 CONCLUSIONS AND RECOMMENDATIONS

1. Data from laboratory tests on a model of the FV weir together with theoretical analysis have been used to predict the attainable height of the prototype weir for the Milk River at Eastern Crossing. The dimensionless weir height $P/H_b$ was found to depend on the relative measured head $h_1/H_b$, the relative grain size $D_{50}/H_b$ and the parameter E which accounts for the viscous properties of the water. The parameter E was found to be important because the flow regime of the model and prototype are in the smooth turbulent flow regime.
2. The results of this study indicate that values of $P/H_b$ for the prototype weir will always be greater than 0.05 for values of relative head $h_1/H_b > 0.43$, which represents flow greater than 5 cms. Although the weir crest will be free of sediment for these

flows, additional study is required to define the behaviour of the discharge coefficient in the range of $0.05 < P/H_b < 0.4$. The behaviour of the discharge coefficient in this range will determine the accuracy of the weir for flows less than 15 cms. Such tests must be conducted with a model large enough to avoid scale effects at the lowest values of $P/H_b$. For values of $P/H_b > 0.4$, the discharge coefficient varies between the values of 0.602 and 0.630. Taking an average value of 0.620 will result in errors of defining the discharge of only 2%. This accuracy applies to flows greater than 15 cms.

3. Analysis indicates that for a weir of given size, values of $P/H_b$ will increase for a given value of $h_1/H_b$, if the median grain size is reduced. Therefore, if the bed material in the prototype approach channel has a grain size smaller than $D_{50}$ = 0.22 mm, used in the model tests, then values of $P/H_b$ in the prototype will be larger than indicated in Figure 23.
4. The present tests were conducted under steady state conditions and do not reflect the effects of unsteady flow during a storm event. During the recession of such flows there may be additional sediment buildup which is not taken into account in this study. There is no reliable way to predict the effect of this. However, it is expected that the weir will clear itself sufficiently at the lower flows to function properly.
5. Available information from the literature indicates that the approach channel is subject to shoaling at the stilling well intake. This does not interfere with the flow characteristics of the weir itself but creates problems in obtaining reliable water level records. An alternative method to determine stage should be sought.
6. It is recommended that a stilling basin be constructed to protect the weir from damage due to excessive scour at the downstream face of the weir block. Tests should be conducted to develop a satisfactory stilling basin.

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**TABLE 1**

Experimental Data

| Run | $h_1$ | $Q$ $m^3/s$ | $P$ cm | $h_1/H_b$ | $P/H_b$ | $D_{50}/H_b$ | $E^{1/3}$ |
|---|---|---|---|---|---|---|---|
| A | 7.37 | 0.0493 | 3.19 | 1.47 | 0.64 | $4.4\times10^{-3}$ | 5.6 |
| B | 10.80 | 0.107 | 5.20 | 2.16 | 1.04 | | |
| C | 5.67 | 0.0257 | 1.53 | 1.13 | 0.31 | | |
| D | 6.53 | 0.0365 | 1.89 | 1.31 | 0.38 | | |
| E | 8 76 | 0.0725 | 3.20 | 1.75 | 0.64 | | |
| F | 10.32 | 0.0997 | 5.37 | 2.06 | 1.07 | | |
| 1* | | | | 0.5 | 0.20 | $4.6\times10^{-3}$ | 7.6 |
| 2* | | | | 1.0 | 0.50 | | |
| 3* | | | | 1.5 | 1.10 | | |
| 10* | | | | 0.5 | -0.14 | $2.2\times10^{-3}$ | 5.6 |
| 11* | | | | 1.0 | 0.15 | | |
| 12* | | | | 1.5 | 0.55 | | |

* Data from White 1971

**TABLE 2**

Values of $\Delta P/H_b$

| Run | $h_1/H_b$ | $P_m/H_b$ | $P_c/H_b$ | $\Delta P/H_b$ | $D_{50}/H_b$ |
|---|---|---|---|---|---|
| A | 1.47 | 0.64 | 0.26 | 0.38 | $4.4 \times 10^{-3}$ |
| B | 2.16 | 1.04 | 1.22 | -0.18 | |
| C | 1.13 | 0.31 | -0.092 | 0.40 | |
| D | 1.31 | 0.38 | 0.080 | 0.30 | |
| E | 1.75 | 0.64 | 0.61 | 0.03 | |
| F | 2.06 | 1.07 | 1.06 | 0.01 | |
| 10 | 0.5 | -0.14 | -0.31 | 0.17 | $2.2 \times 10^{-3}$ |
| 11 | 1.0 | 0.15 | -0.19 | 0.34 | |
| 12 | 1.5 | 0.55 | 0.29 | 0.26 | |

$\Delta P = P_m - P_c$

m and c denote measured and computed

**TABLE 3**

Adjusted Prototype Value of Relative Weir Height Using $\Delta P/H_b$

| $h_1/H_b$ | $P_c/H_b$ | $\Delta P/H_b$ | $(P/H_b)_p$ |
|---|---|---|---|
| 0.2 | -0.17 | 0.07 | -0.10 |
| 0.4 | -0.13 | 0.14 | 0.01 |
| 0.6 | 0.08 | 0.21 | 0.29 |
| 0.8 | 0.52 | 0.28 | 0.80 |
| 1.0 | 1.20 | 0.35 | 1.55 |
| 1.25 | 2.35 | 0.35 | 2.70 |
| 1.50 | 3.74 | 0.27 | 4.01 |

$(P/H_b)_p = P_c/H_b + \Delta P/H_b$

**TABLE 4**

Effect of Reducing Grainsize on Weir Height

| | $D_{50}$ = 0.22 m | | | $D_{50}$ = 0.15 mm | | |
|---|---|---|---|---|---|---|
| $h_1/H_b$ | $D_{50}/H_b$ $x10^{-4}$ | $P_c/H_b$ | $E^{1/3}$ | $D_{50}/H_b$ $x10^{-4}$ | $P_c/H_b$ | $E^{1/3}$ |
| 0.2 | 1.8 | -0.14 | 5.6 | 1.25 | -0.14 | 3.8 |
| 0.4 | | -0.13 | | | -0.12 | |
| 0.6 | | 0.08 | | | 0.11 | |
| 0.8 | | 0.52 | | | 0.57 | |
| 1.0 | | 1.20 | | | 1.28 | |
| 1.2 | | 2.35 | | | 2.50 | |

Note: Values of $P_c/H_b$ are predicted values for the prototype and have note been adjusted with measured data. They are for relative comparison only.

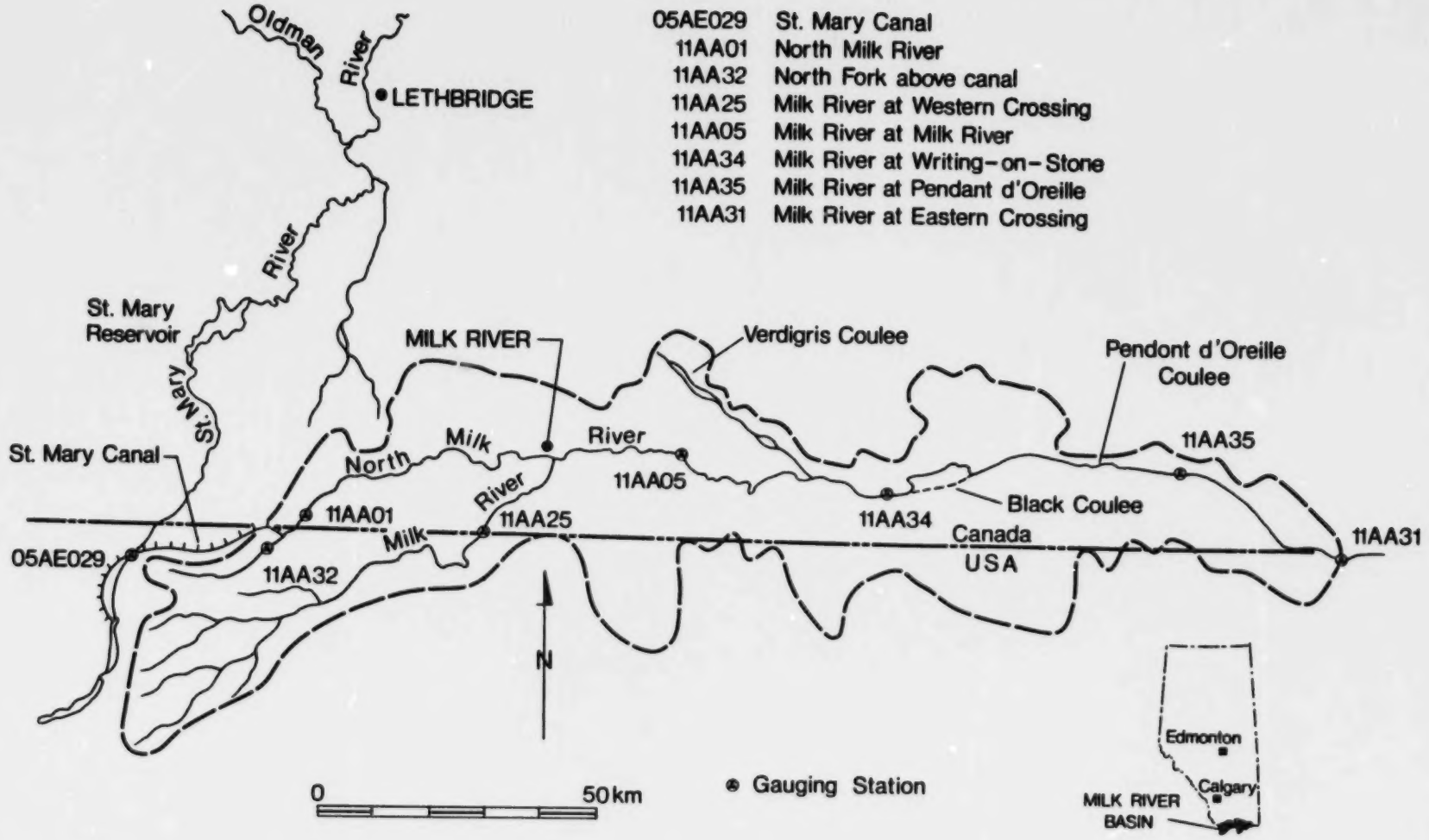


FIGURE 1. Milk River basin

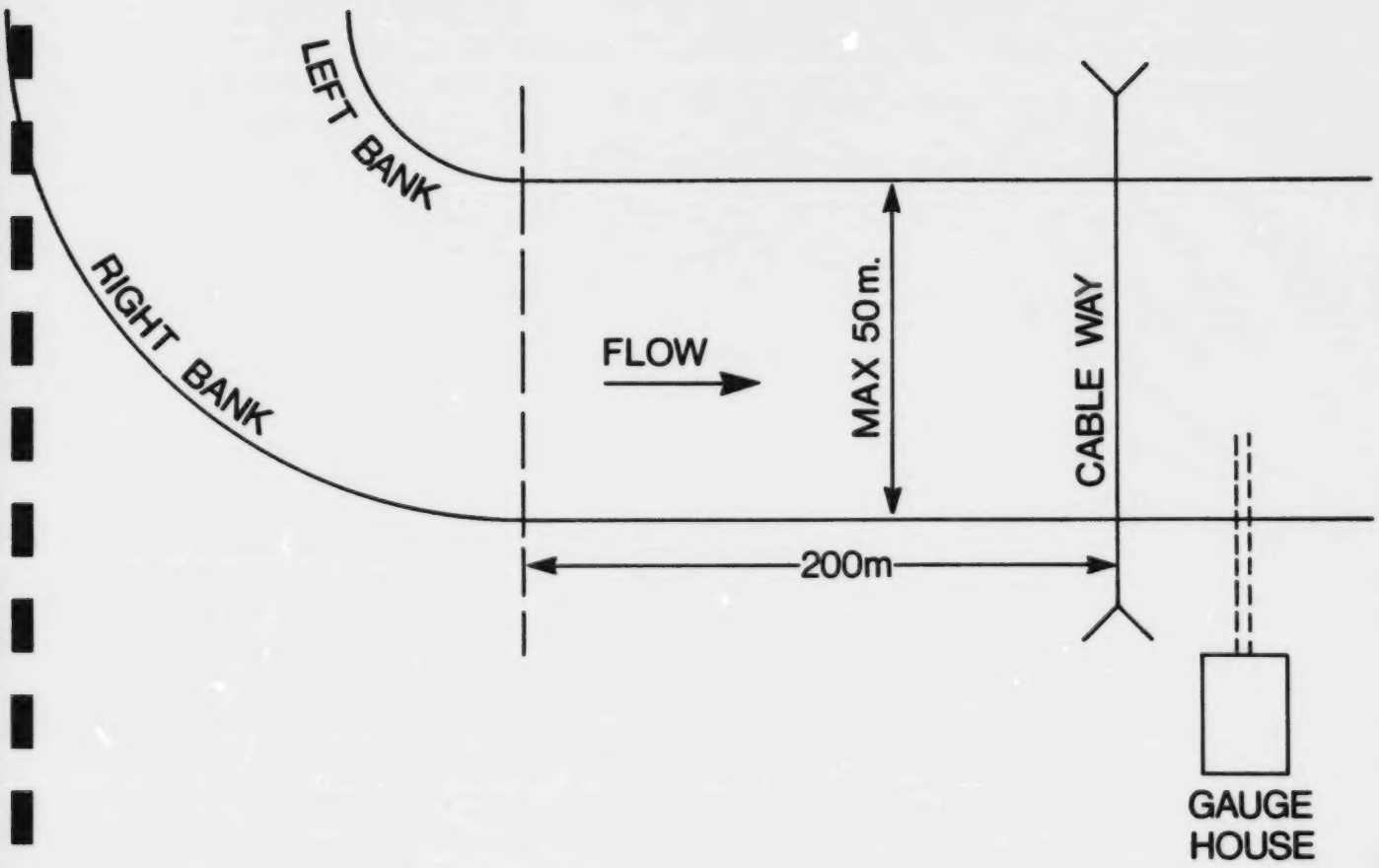


FIGURE 2. Approximate site plan at eastern crossing

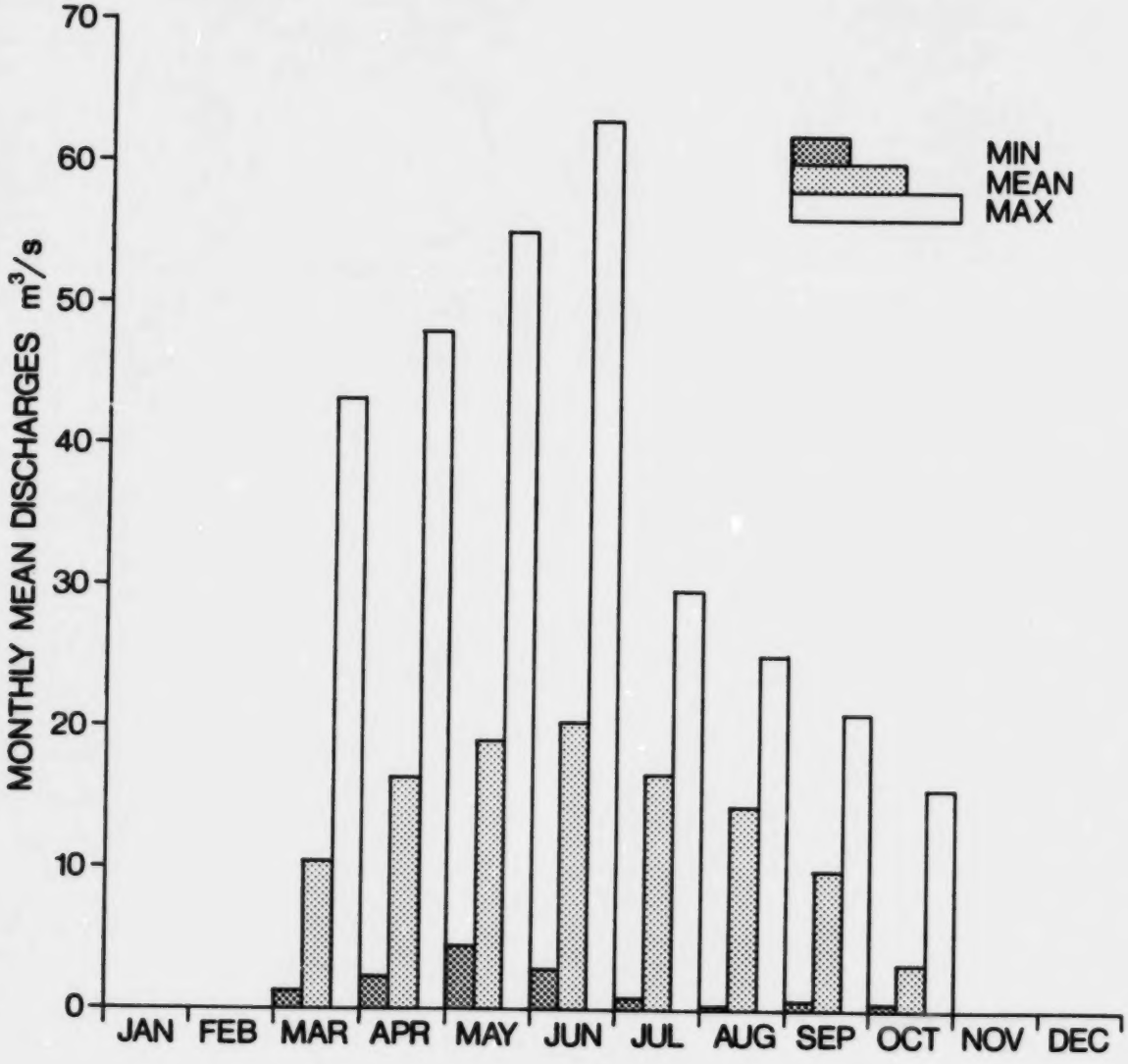


FIGURE 3. Distribution of monthly mean flows, for period of record.

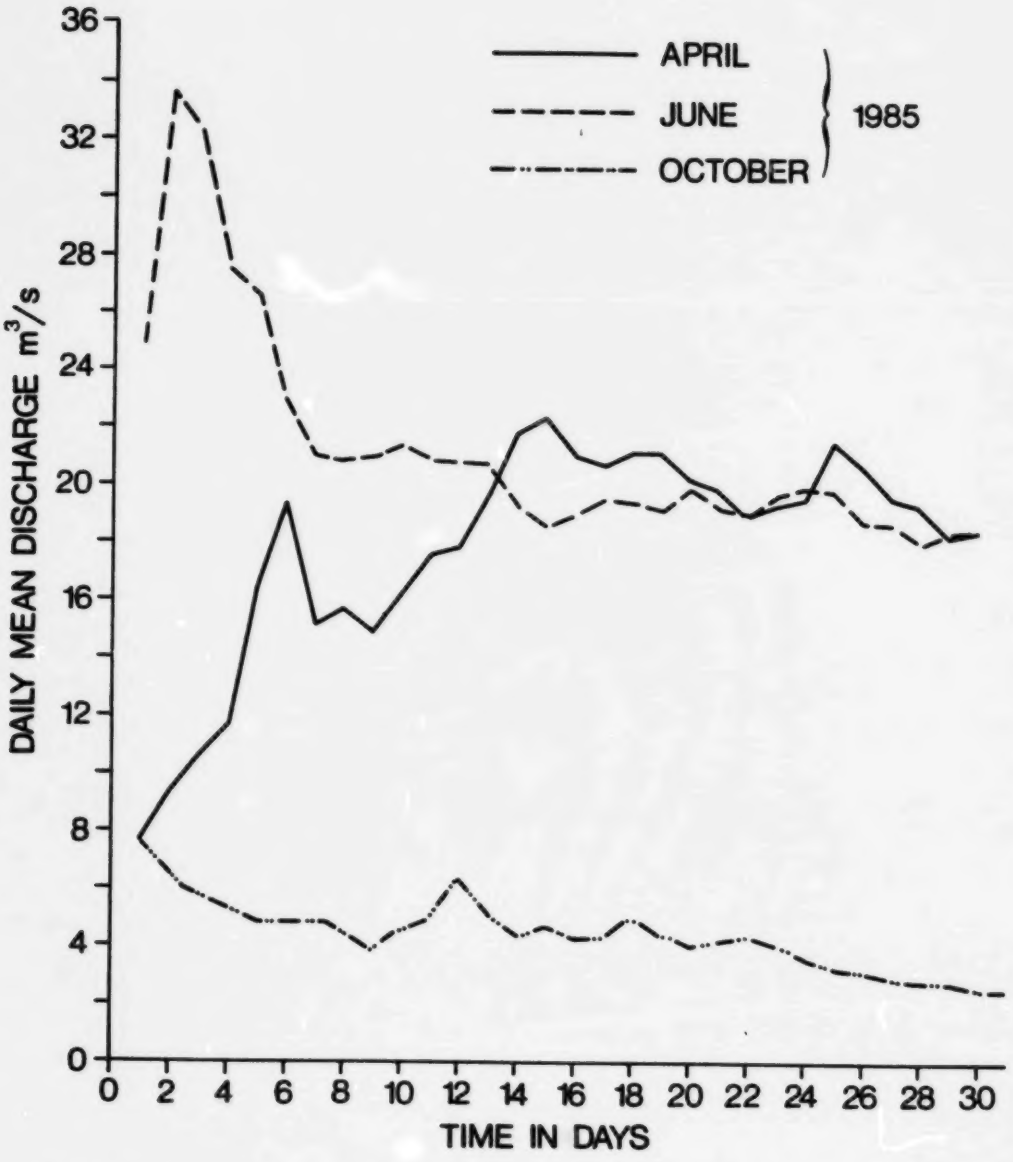


FIGURE 4. Hydrographs for daily mean flows.

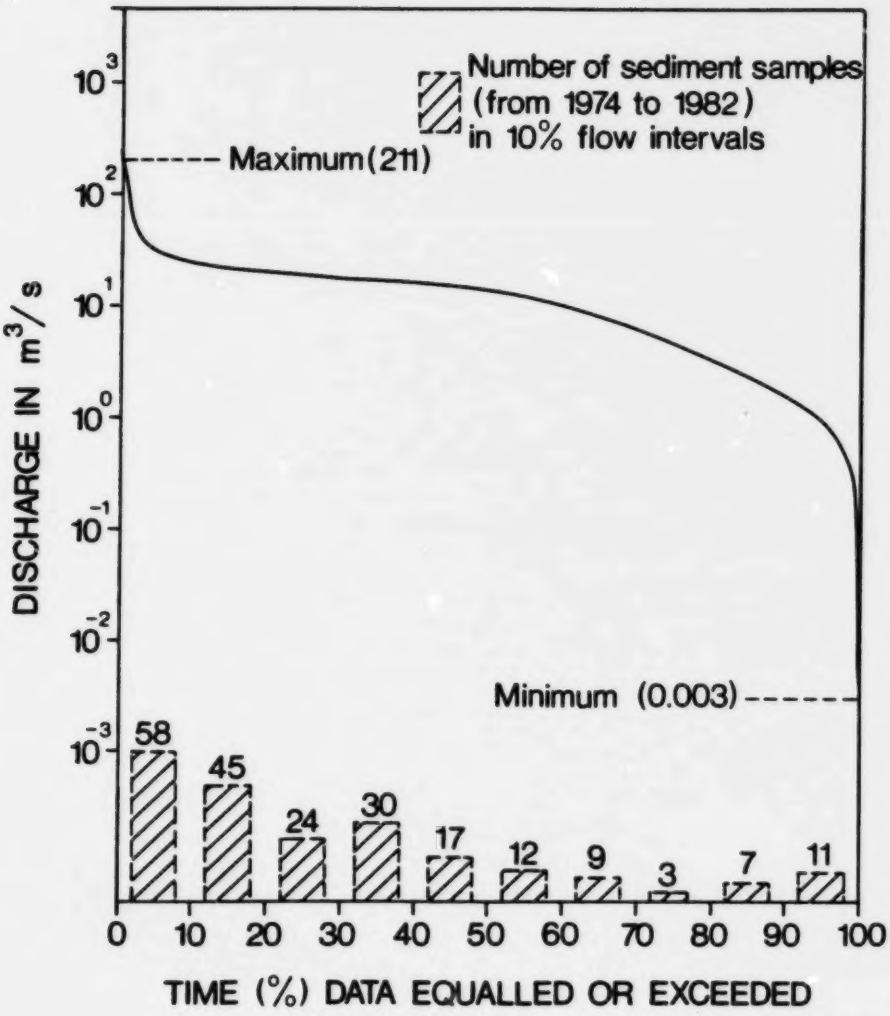


FIGURE 5. Flow duration at eastern crossing (W.R.B. 1986)

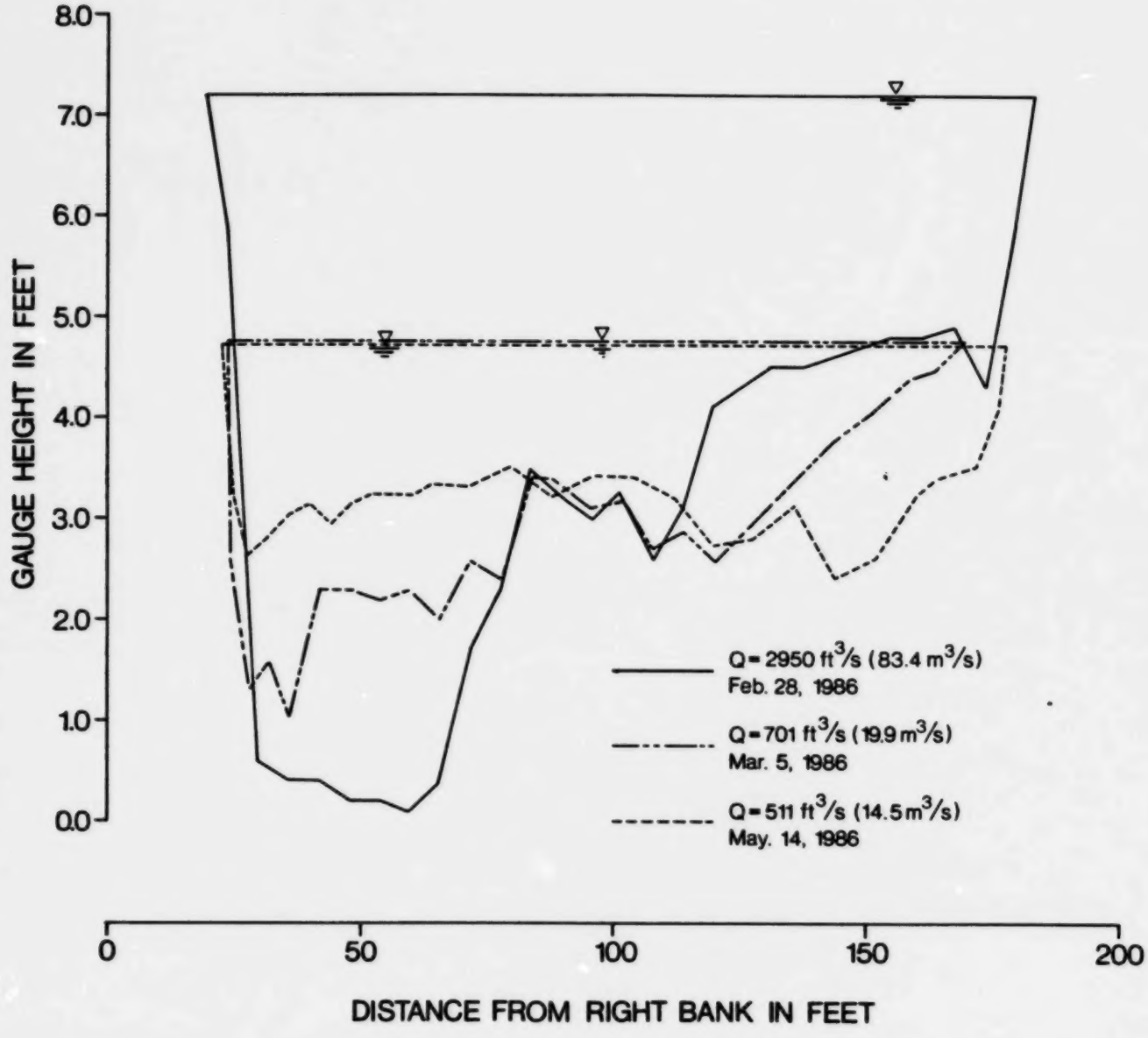


FIGURE 6. Variability of crossection

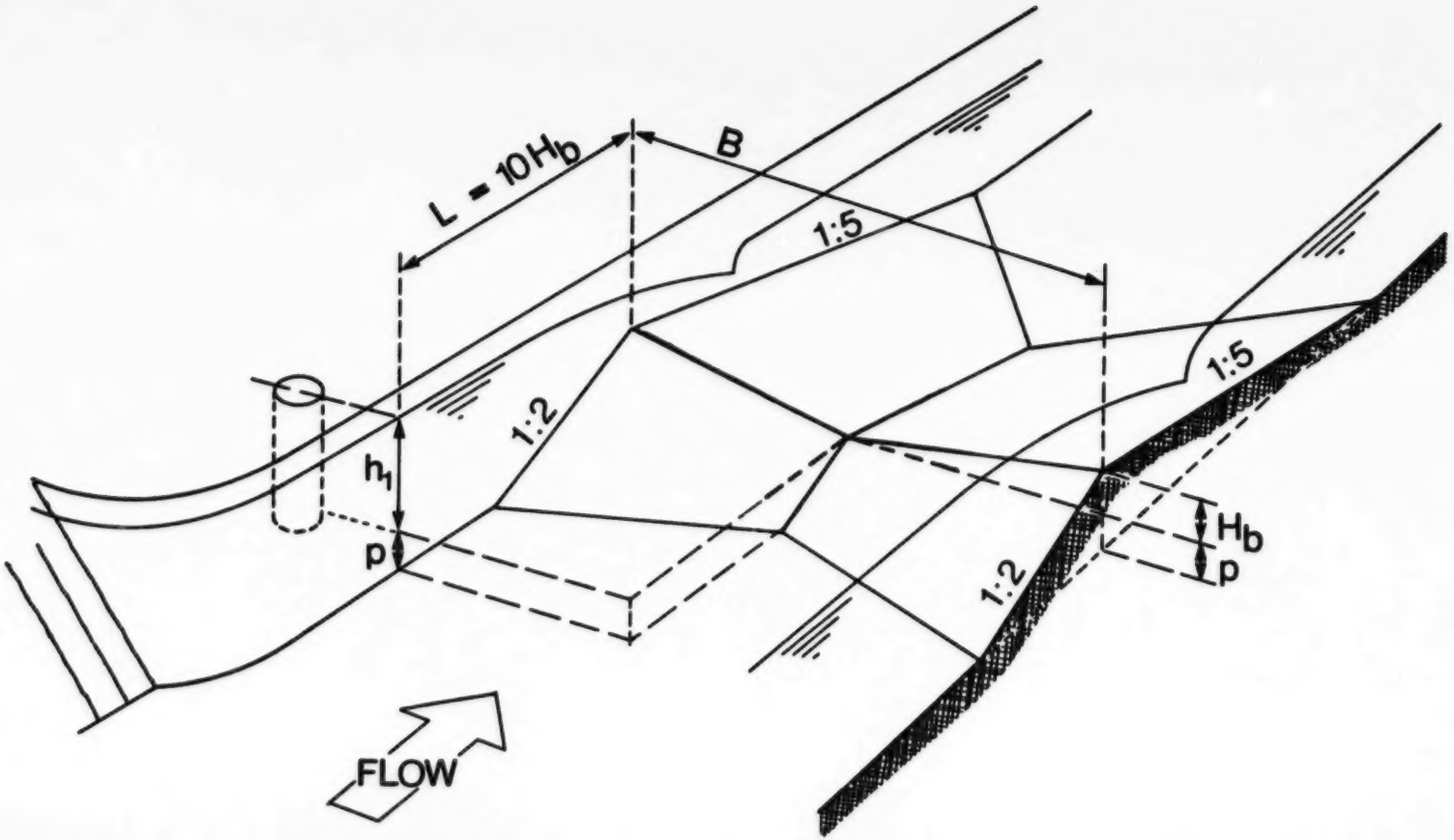


FIGURE 7. Triangular profile flat – V weir.

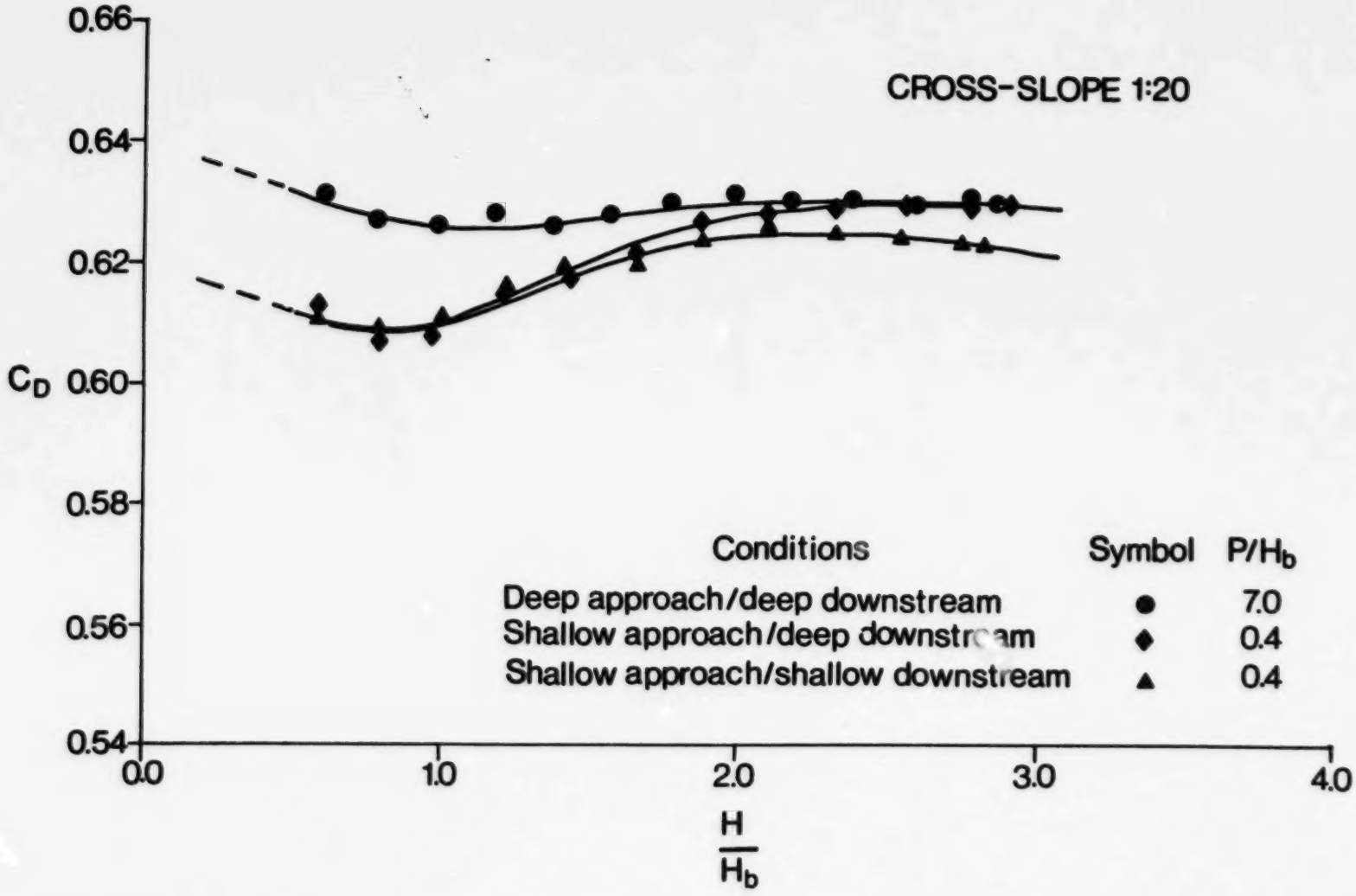


FIGURE 8. Effect of $P/H_b$ on the Coefficient of Discharge (White, 1970)

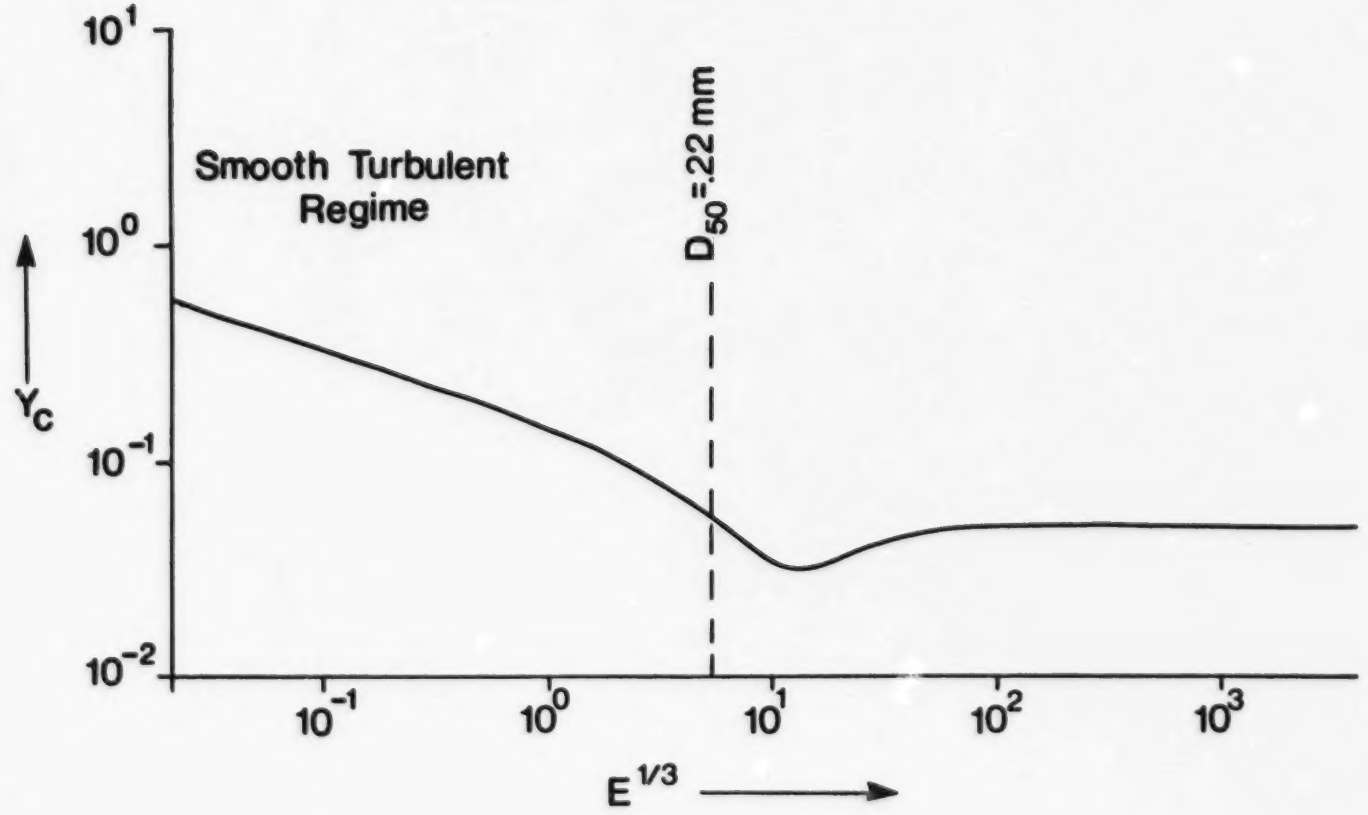


FIGURE 9. Sediment Transport Inception Curve (Yalin et al, 1988)

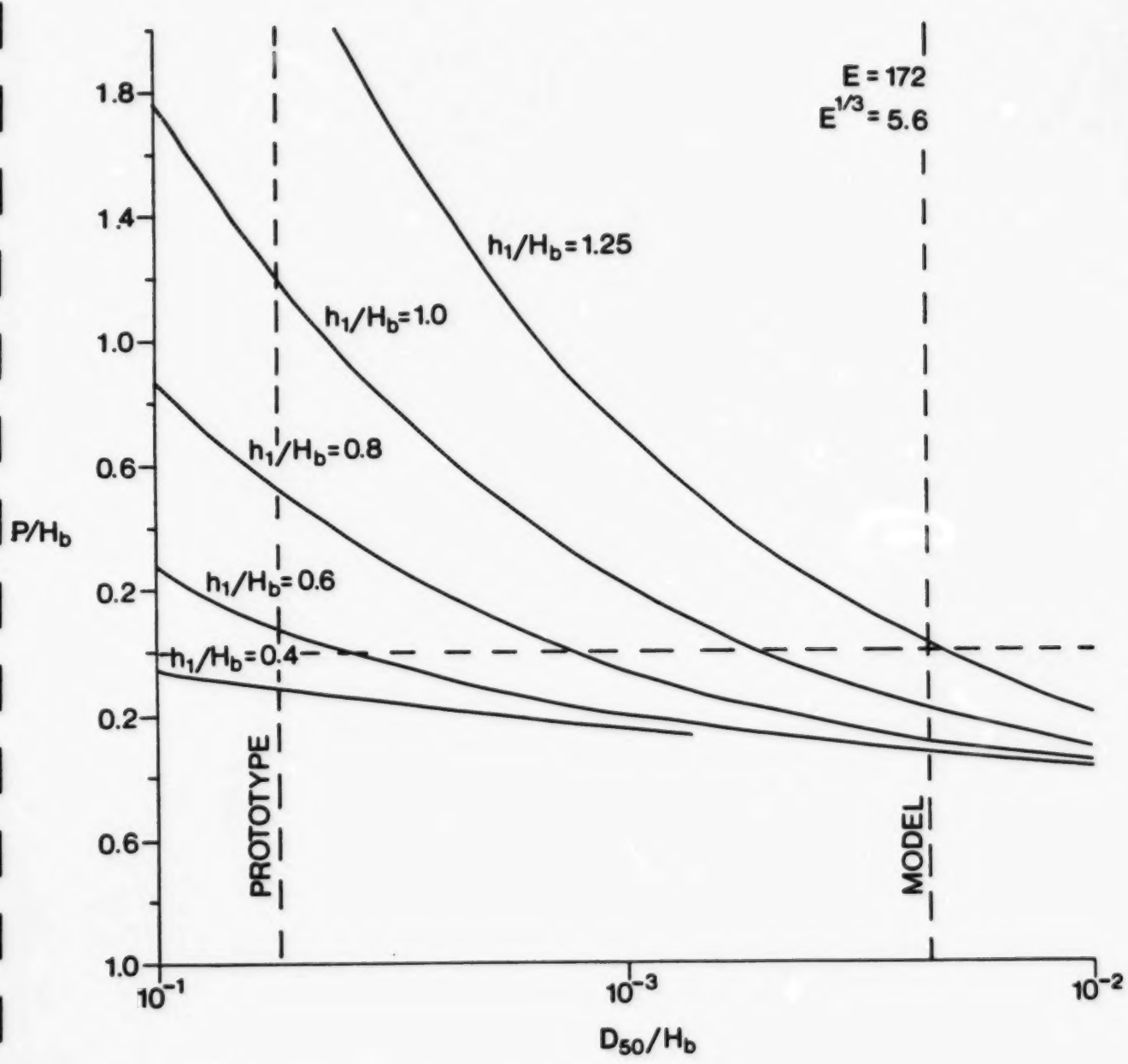


FIGURE 10. Predicted Variability of $P/H_b$

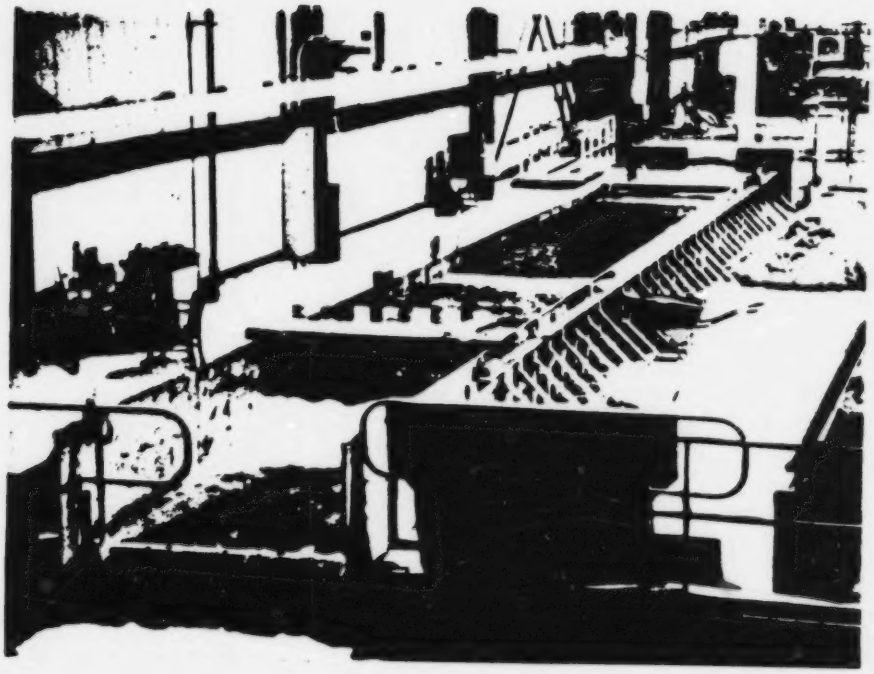

FIGURE 11. Sediment Flume used in Model Tests

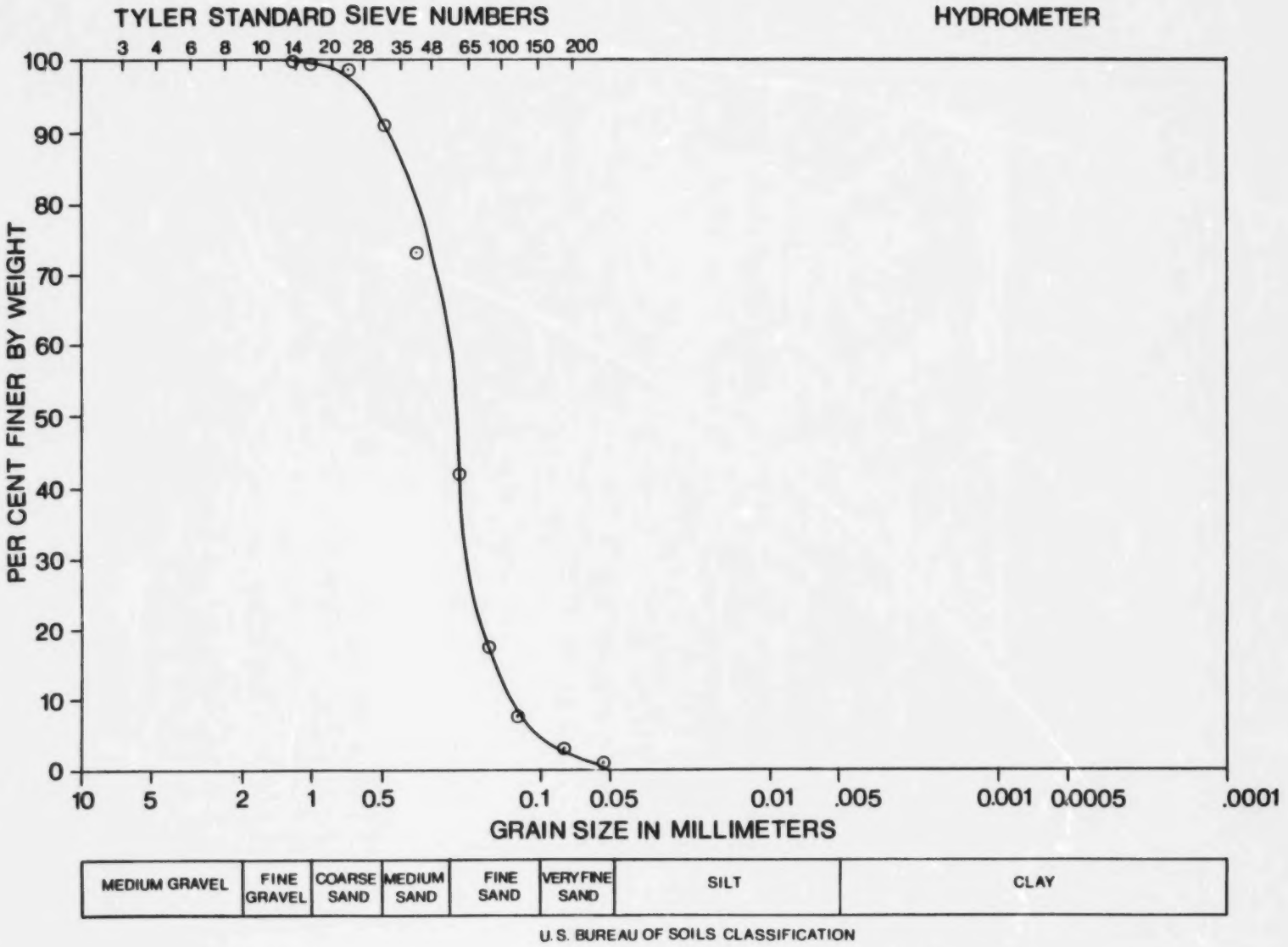


FIGURE 12. Grainsize Distribution for Test Sand

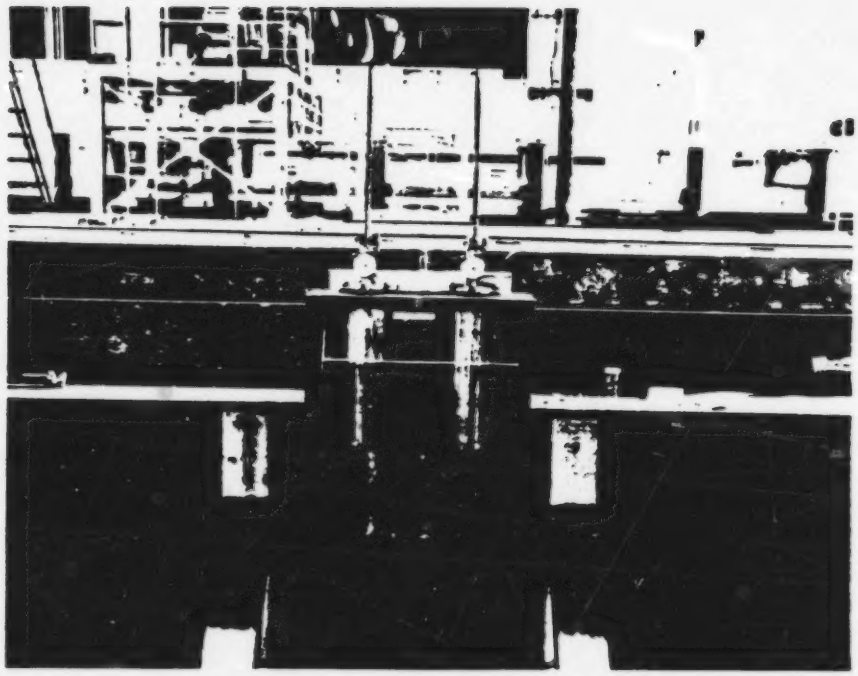

FIGURE 13. Stilling Wells and Point Gauges

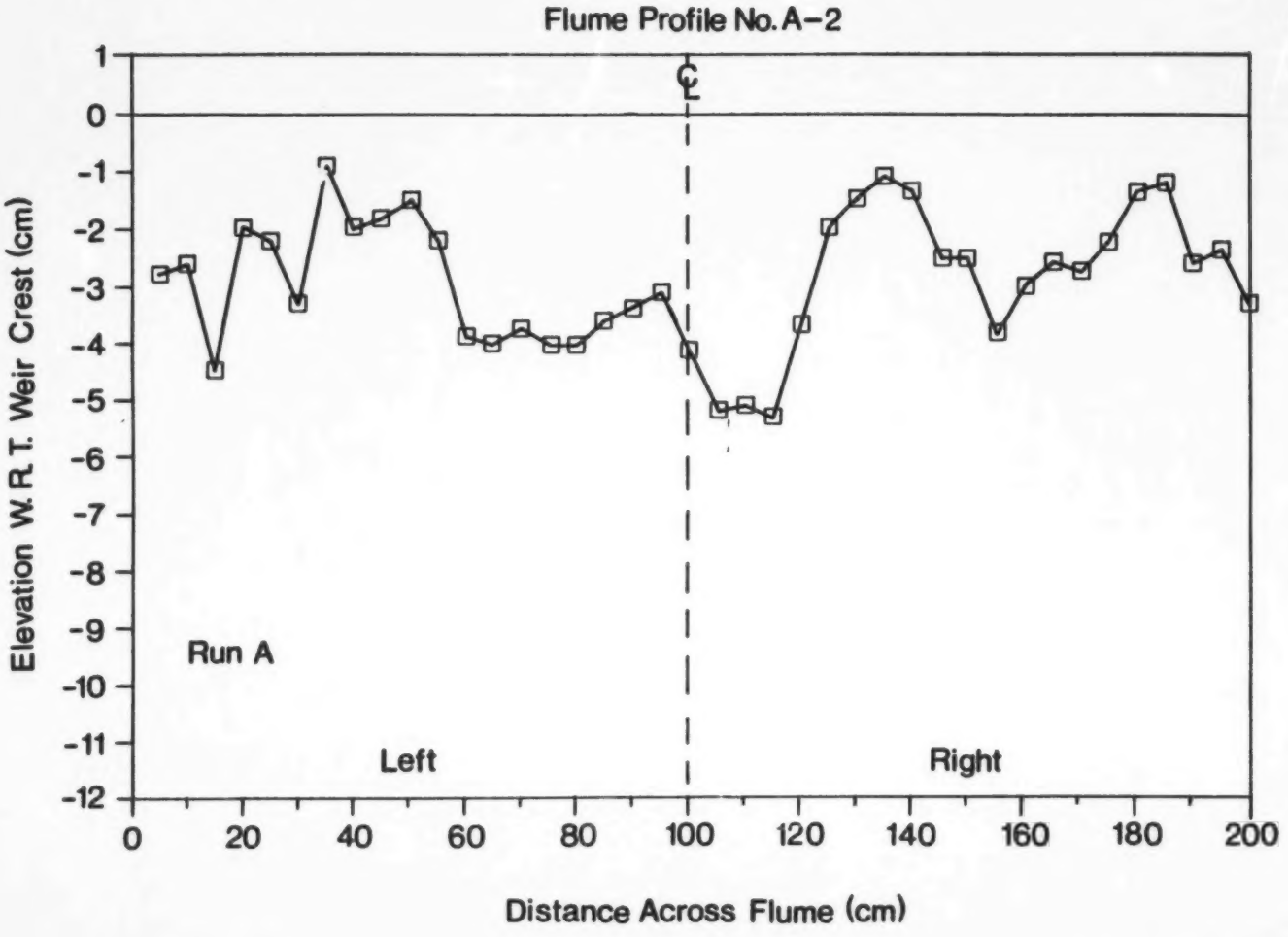


FIGURE 14. Bed Cross Section at Stilling Well Intake

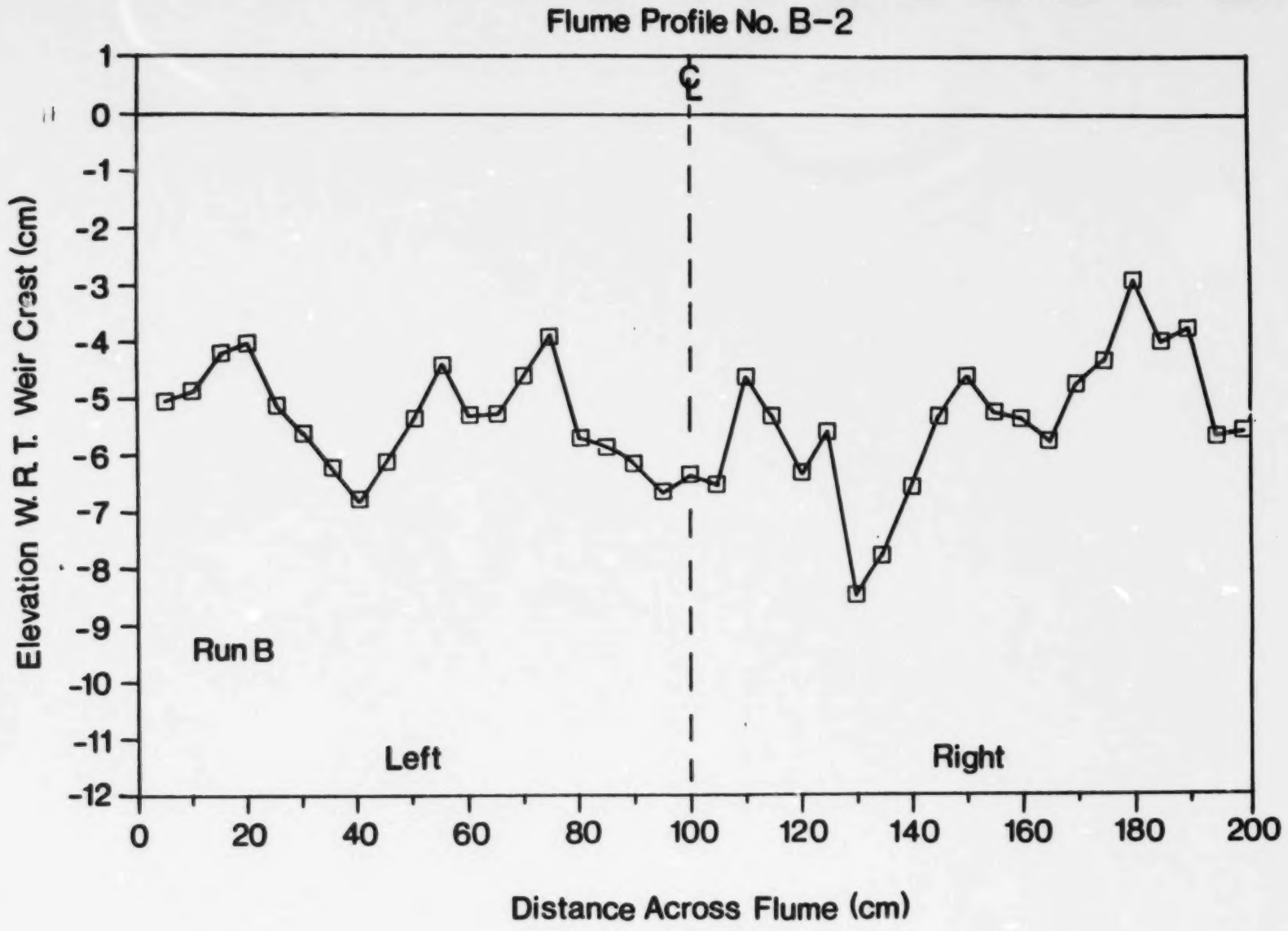


FIGURE 15. Bed Cross Section at Stilling Well Intake

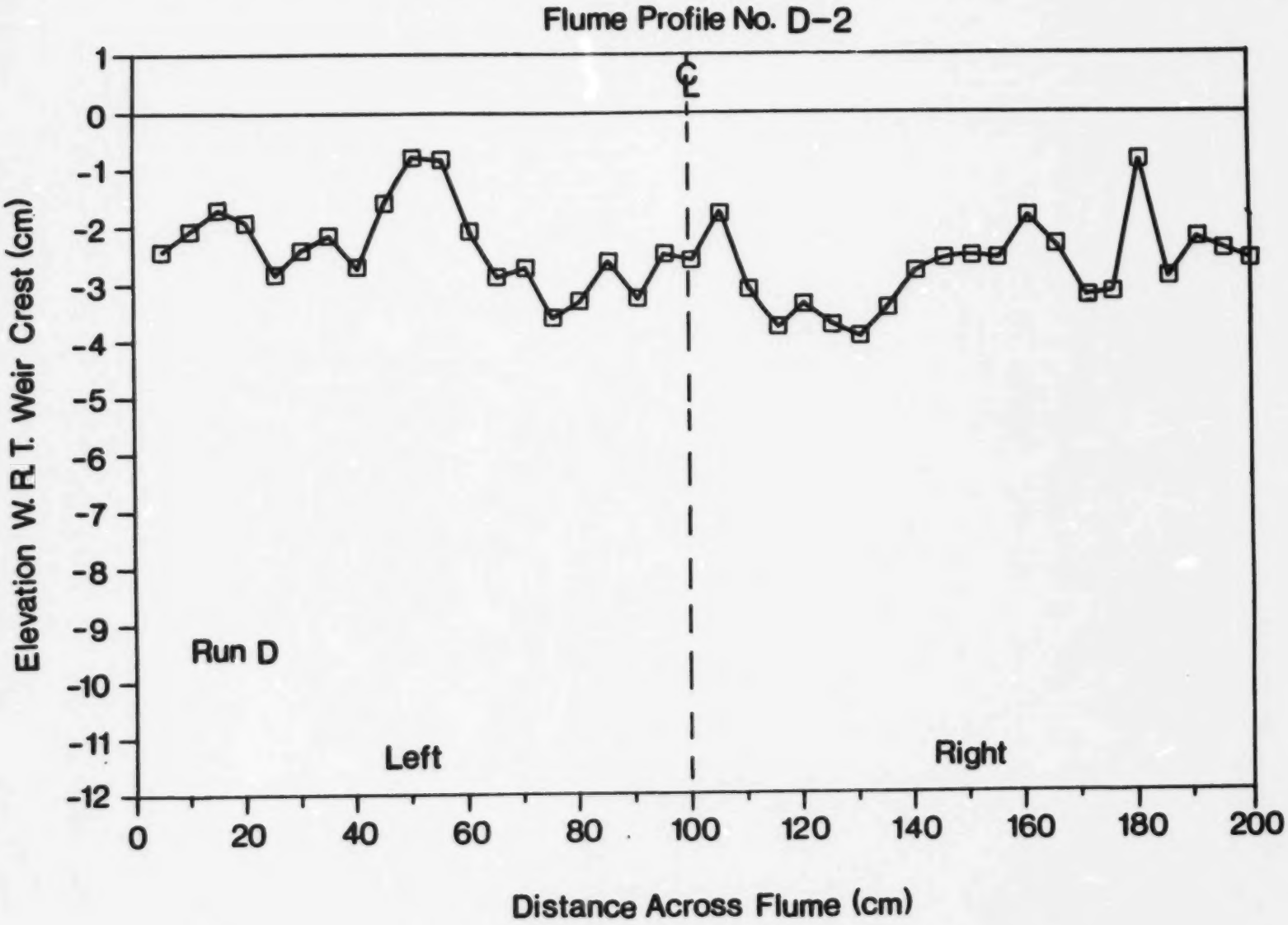


FIGURE 16. Bed Cross Section at Stilling Well Intake

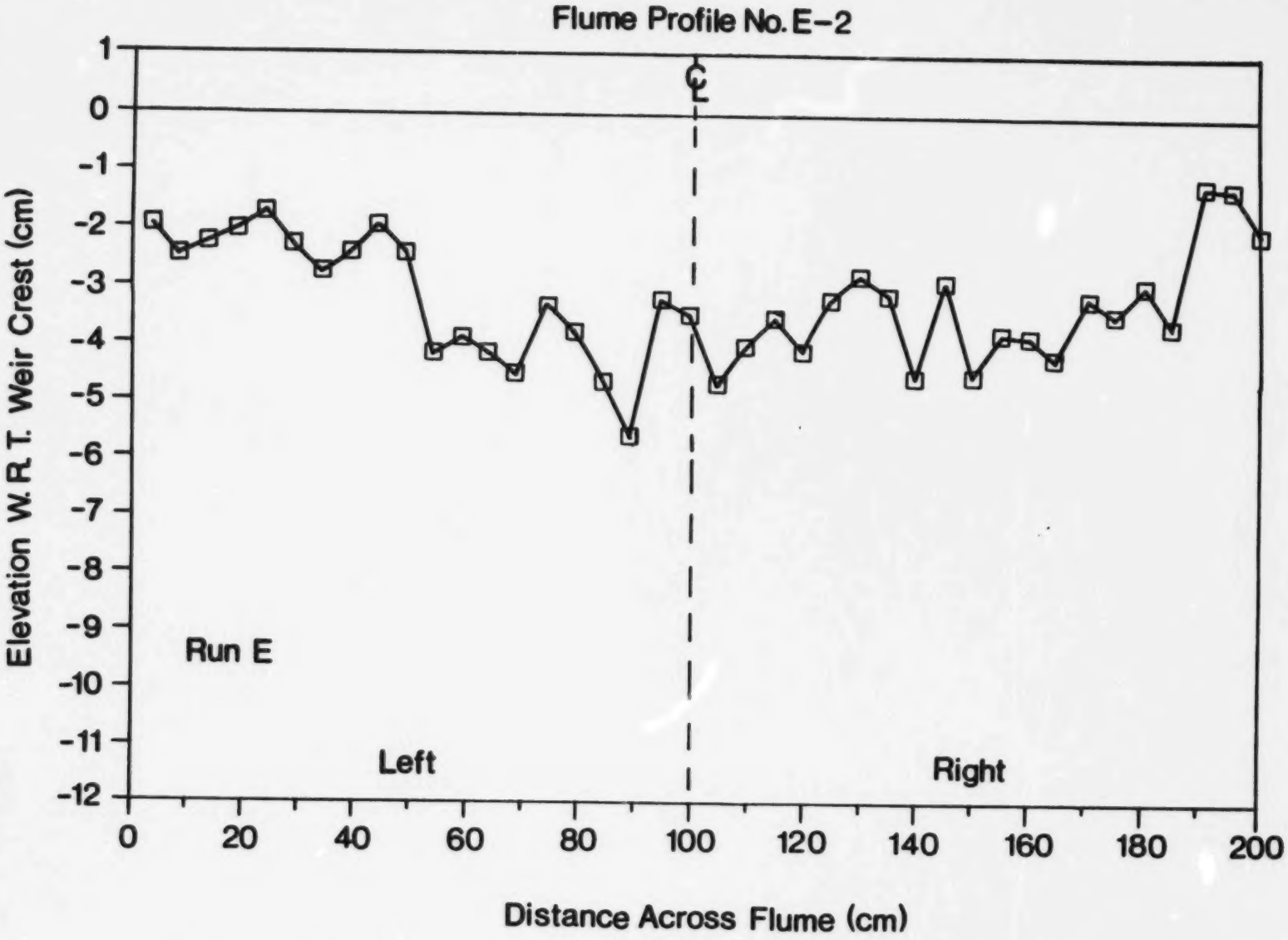


FIGURE 17. Bed Cross Section at Stilling Well Intake

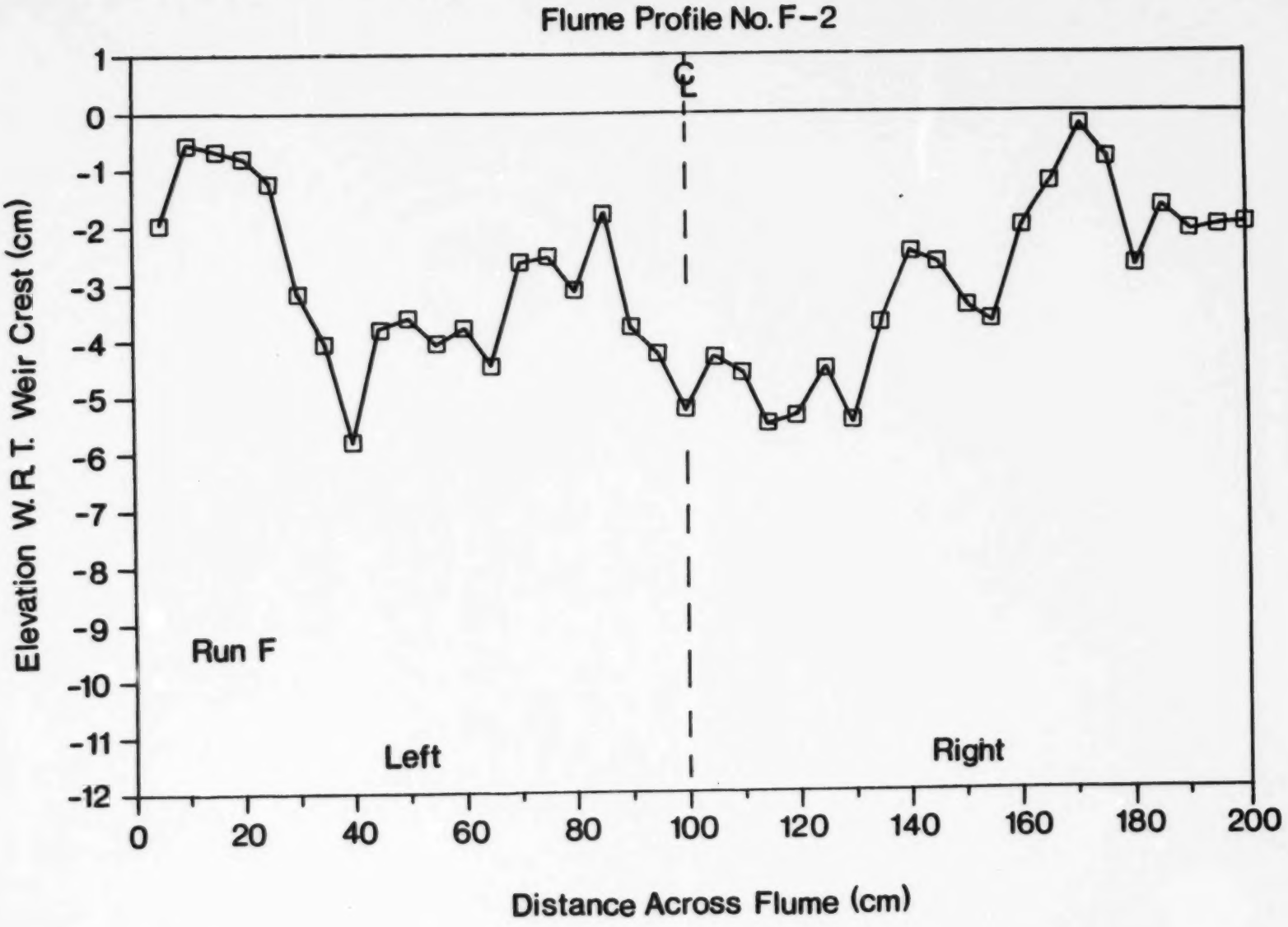


FIGURE 18. Bed Cross Section at Stilling Well Intake

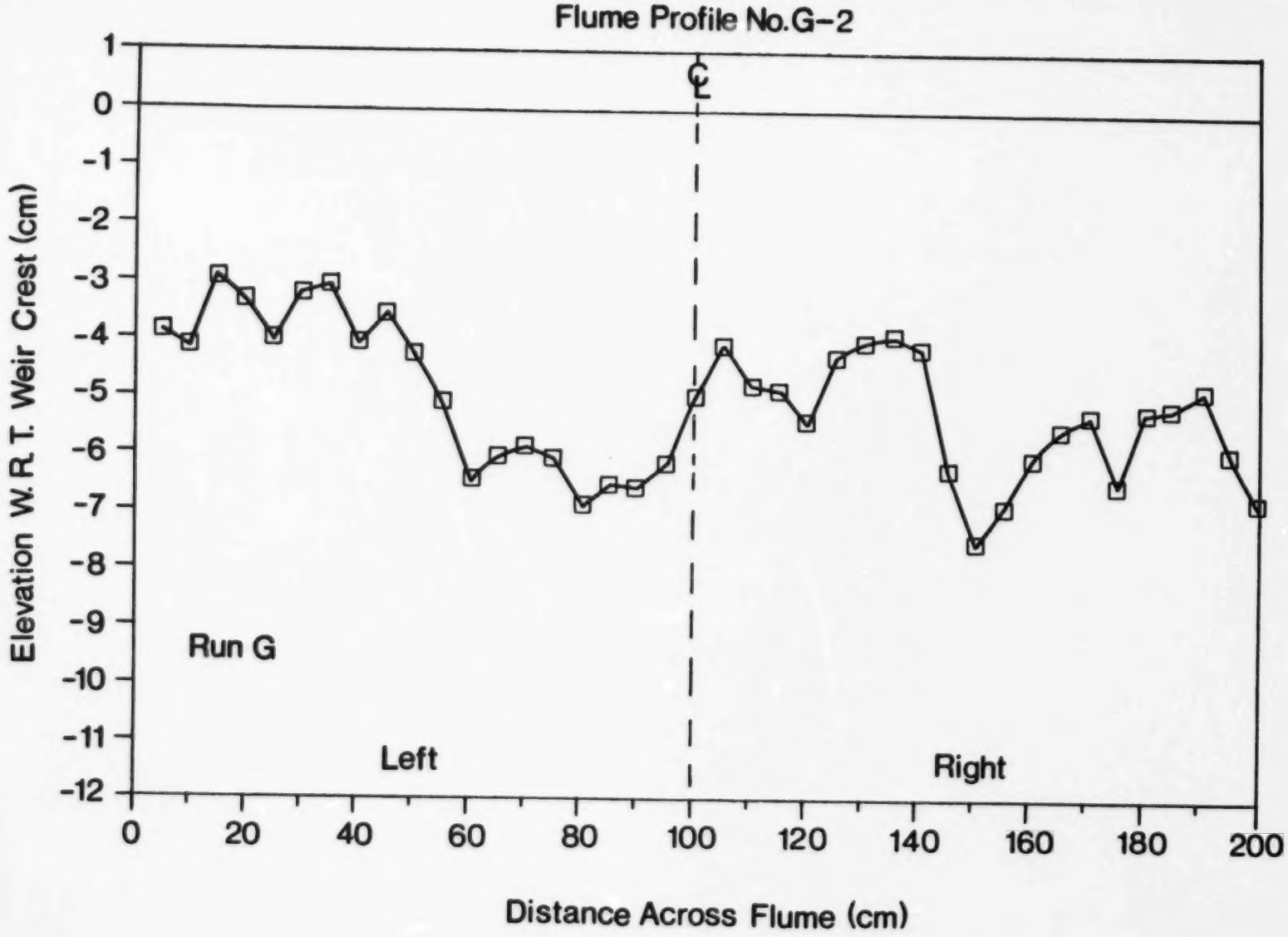


FIGURE 19. Bed Cross Section at Stilling Well Intake

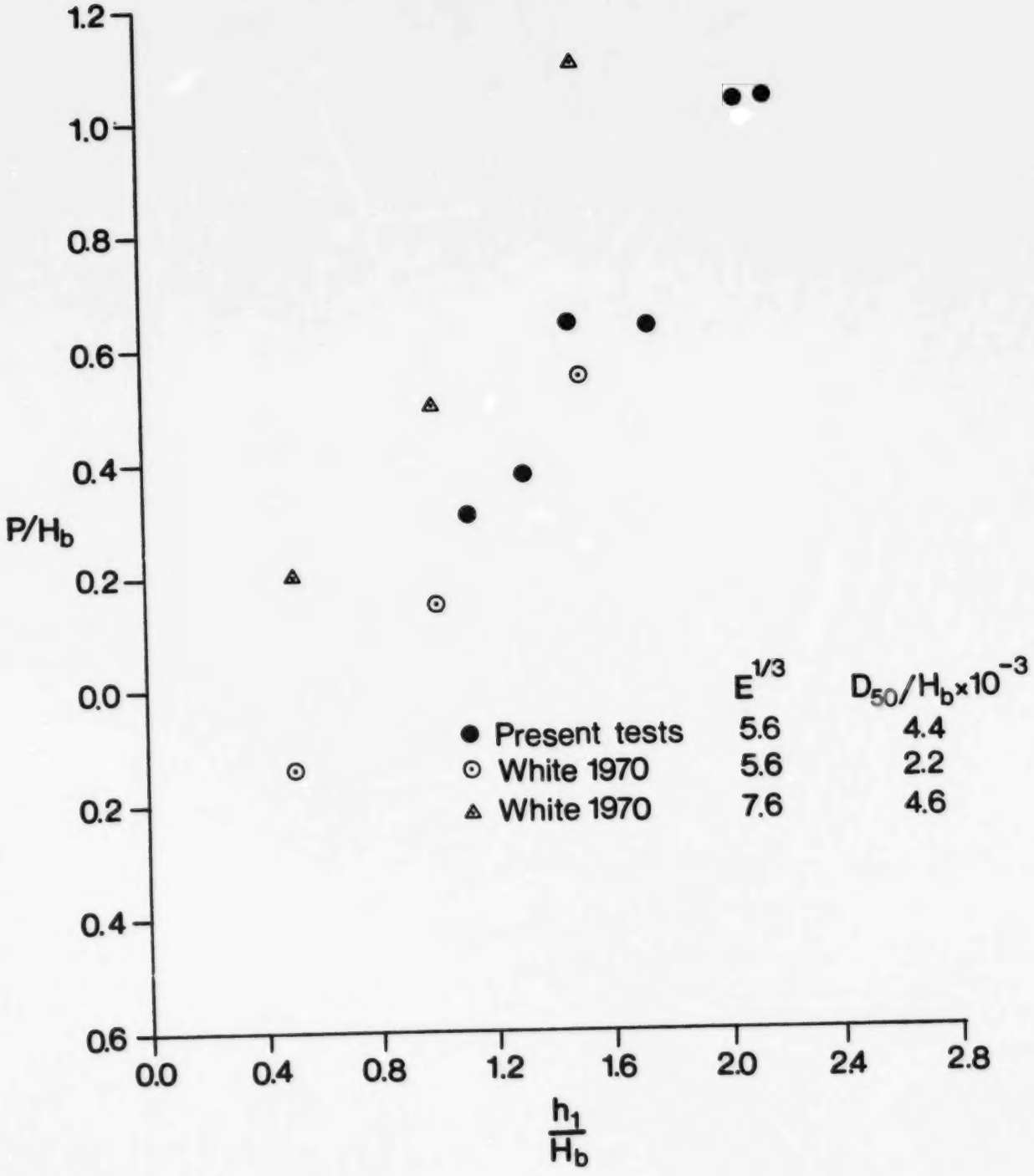


FIGURE 20. Dimensionless Plot of Test Data

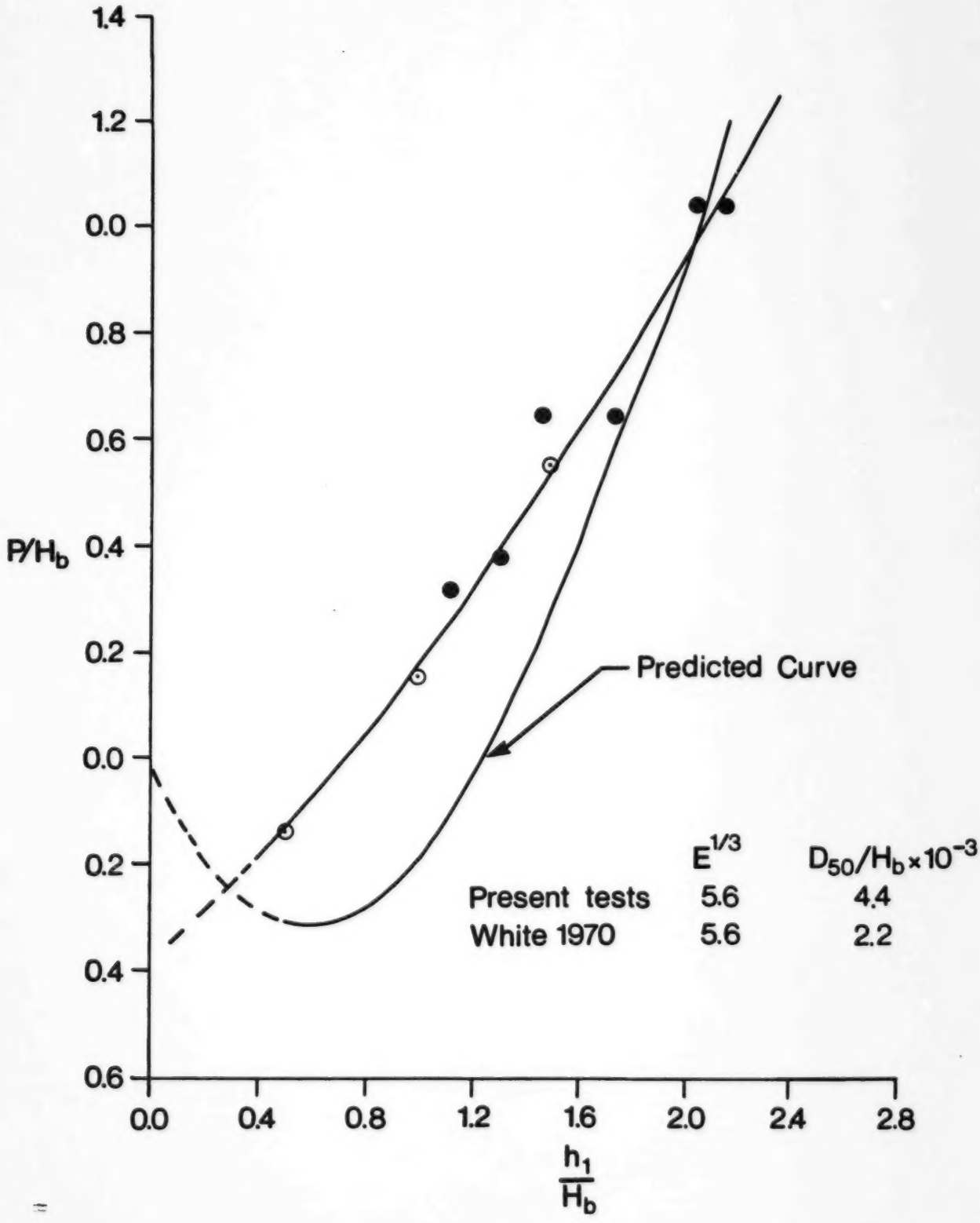


FIGURE 21. Experimental and Predicted Values of $P/H_b$

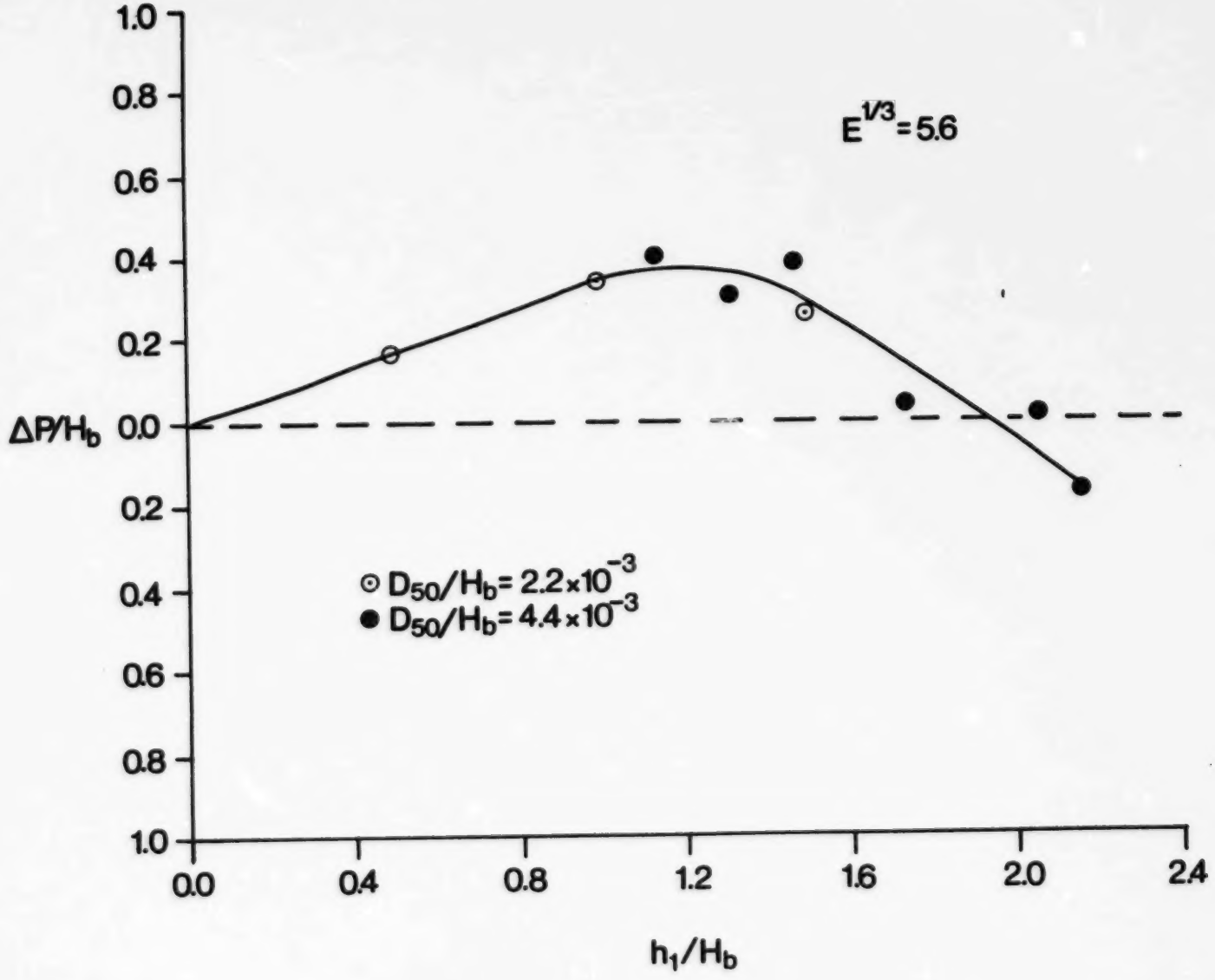


FIGURE 22. Variation of $\Delta P/H_b$

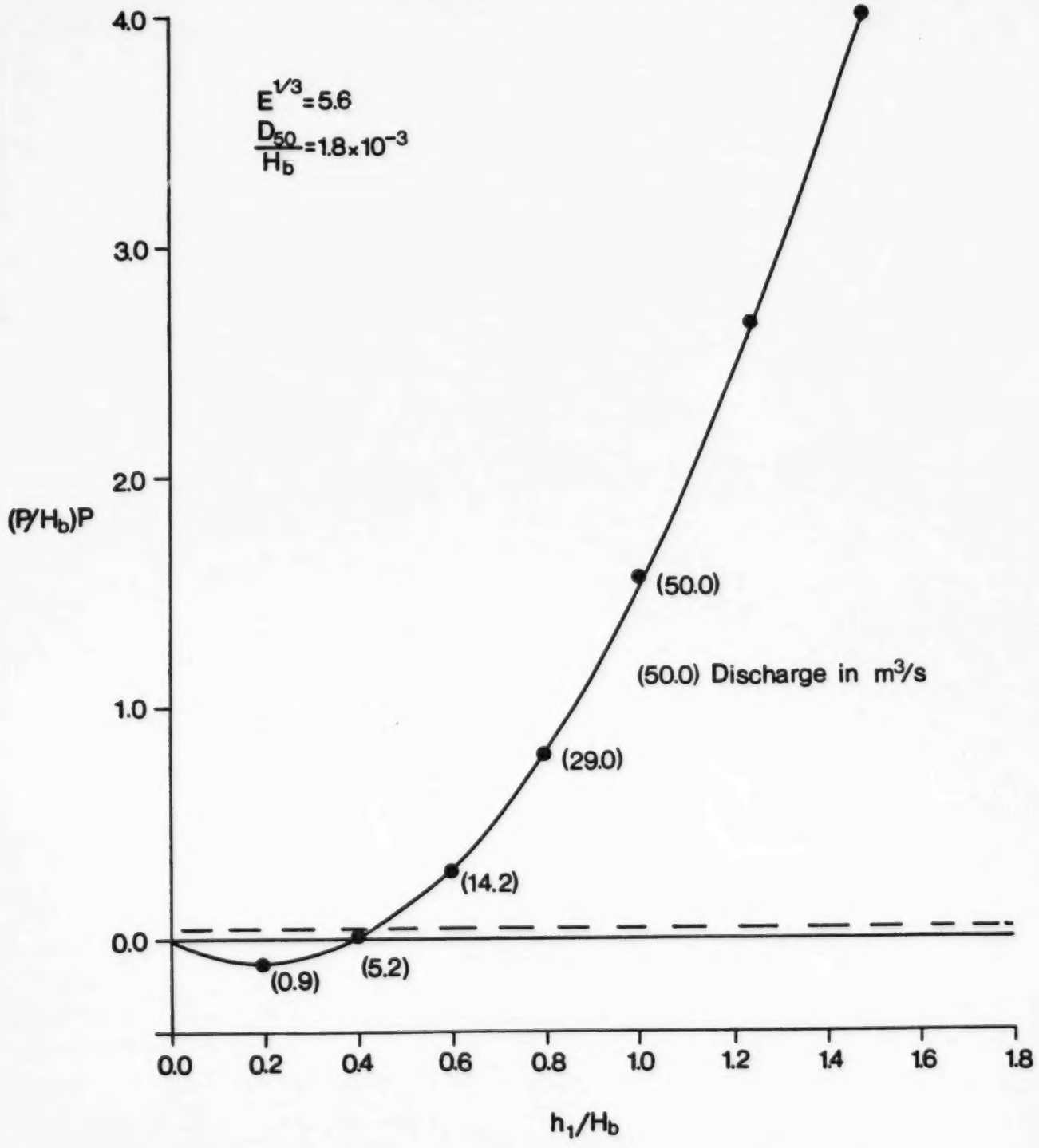


FIGURE 23. Values of $P/H_b$ for Prototype Weir